Texas
Educator Certification
Examination Program

Preparation Manual
Mathematics 4–8 (115)

Overview and Exam Framework

Reference Materials

Sample Selected-Response Questions

Sample Selected-Response Answers and Rationales

Section 3: Overview and Exam Framework
Mathematics 4–8 (115)

Exam Overview

Exam Name	Mathematics 4–8
Exam Code	115
Time	5 hours
Number of Questions	100 selected-response questions
Format	Computer-administered test (CAT)

The TExES Mathematics 4–8 (115) exam is designed to assess whether an examinee has the requisite knowledge and skills that an entry-level educator in this field in Texas public schools must possess. The 100 selected-response questions are based on the Mathematics 4–8 exam framework and cover grades 4–8. The exam may contain questions that do not count toward the score. Your final scaled score will be based only on scored questions.

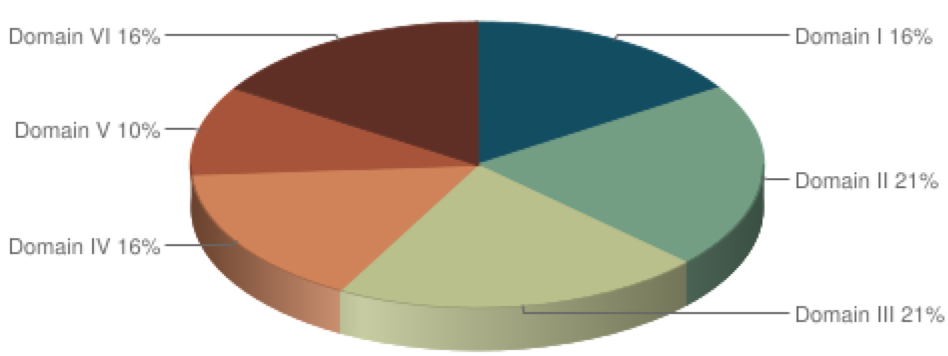
The Standards

Standard I	Number Concepts: The mathematics teacher understands and uses numbers, number systems and their structure, operations and algorithms, quantitative reasoning and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) to prepare students to use mathematics.
Standard II	Patterns and Algebra: The mathematics teacher understands and uses patterns, relations, functions, algebraic reasoning, analysis and technology appropriate to teach the statewide curriculum (TEKS) to prepare students to use mathematics.
Standard III	Geometry and Measurement: The mathematics teacher understands and uses geometry, spatial reasoning, measurement concepts and principles and technology appropriate to teach the statewide curriculum (TEKS) to prepare students to use mathematics.
Standard IV	Probability and Statistics: The mathematics teacher understands and uses probability and statistics, their applications and technology appropriate to teach the statewide curriculum (TEKS) to prepare students to use mathematics.
Standard V	Mathematical Processes: The mathematics teacher understands and uses mathematical processes to reason mathematically, to solve mathematical problems, to make mathematical connections within and outside of mathematics and to communicate mathematically.

Standard VI	Mathematical Perspectives: The mathematics teacher understands the historical development of mathematical ideas, the relationship between society and mathematics, the structure of mathematics and the evolving nature of mathematics and mathematical knowledge.
Standard VII	Mathematical Learning and Instruction: The mathematics teacher understands how children learn and develop mathematical skills, procedures and concepts; knows typical errors students make; and uses this knowledge to plan, organize and implement instruction to meet curriculum goals and to teach all students to understand and use mathematics.
Standard VIII	Mathematical Assessment: The mathematics teacher understands assessment, and uses a variety of formal and informal assessment techniques appropriate to the learner on an ongoing basis to monitor and guide instruction and to evaluate and report student progress.

Domains and Competencies

Domain	Domain Title	Approx. Percentage of Exam	Standards Assessed
I	Number Concepts	16%	Mathematics I
II	Patterns and Algebra	21%	Mathematics II
III	Geometry and Measurement	21%	Mathematics III
IV	Probability and Statistics	16%	Mathematics IV
V	Mathematical Processes and Perspectives	10%	Mathematics V–VI
VI	Mathematical Learning, Instruction and Assessment	16%	Mathematics VII–VIII



The content covered by this exam is organized into broad areas of content called **domains**. Each domain covers one or more of the educator standards for this field. Within each domain, the content is further defined by a set of

competencies. Each competency is composed of two major parts:

- The **competency statement**, which broadly defines what an entry-level educator in this field in Texas public schools should know and be able to do.
- The **descriptive statements**, which describe in greater detail the knowledge and skills eligible for testing.

Domain I—Number Concepts

Competency 001—The teacher understands the structure of number systems, the development of a sense of quantity and the relationship between quantity and symbolic representations.

The beginning teacher:

A. Analyzes the structure of numeration systems and the roles of place value and zero in the base ten system.

B. Understands the relative magnitude of whole numbers, integers, rational numbers and real numbers.

C. Demonstrates an understanding of a variety of models for representing numbers (e.g., fraction strips, diagrams, patterns, shaded regions, number lines).

D. Demonstrates an understanding of equivalency among different representations of rational numbers.

E. Selects appropriate representations of real numbers (e.g., fractions, decimals, percents, roots, exponents, scientific notation) for particular situations.

F. Understands the characteristics of the set of whole numbers, integers, rational numbers, real numbers and complex numbers (e.g., commutativity, order, closure, identity elements, inverse elements, density).

G. Demonstrates an understanding of how some situations that have no solution in one number system (e.g., whole numbers, integers, rational numbers) have solutions in another number system (e.g., real numbers, complex numbers).

Competency 002—The teacher understands number operations and computational algorithms.

The beginning teacher:

A. Works proficiently with real and complex numbers and their operations.

B. Analyzes and describes relationships between number properties, operations and algorithms for the four basic operations involving integers, rational numbers and real numbers.

C. Uses a variety of concrete and visual representations to demonstrate the connections between operations and algorithms.

D. Justifies procedures used in algorithms for the four basic operations with integers, rational numbers and real numbers and analyzes error patterns that may occur in their application.

E. Relates operations and algorithms involving numbers to algebraic procedures (e.g., adding fractions to adding rational expressions, division of integers to division of polynomials).

F. Extends and generalizes the operations on rationals and integers to include exponents, their properties and their applications to the real numbers.

Competency 003—The teacher understands ideas of number theory and uses numbers to model and solve problems within and outside of mathematics.

The beginning teacher:

A. Demonstrates an understanding of ideas from number theory (e.g., prime factorization, greatest common divisor) as they apply to whole numbers, integers and rational numbers and uses these ideas in problem situations.

B. Uses integers, rational numbers and real numbers to describe and quantify phenomena such as money, length, area, volume and density.

C. Applies knowledge of place value and other number properties to develop techniques of mental mathematics and computational estimation.

D. Applies knowledge of counting techniques such as permutations and combinations to quantify situations and solve problems.

E. Applies properties of the real numbers to solve a variety of theoretical and applied problems.

Domain II—Patterns and Algebra

Competency 004—The teacher understands and uses mathematical reasoning to identify, extend and analyze patterns and understands the relationships among variables, expressions, equations, inequalities, relations and functions.

The beginning teacher:

A. Uses inductive reasoning to identify, extend and create patterns using concrete models, figures, numbers and algebraic expressions.

B. Formulates implicit and explicit rules to describe and construct sequences verbally, numerically, graphically and symbolically.

C. Makes, tests, validates and uses conjectures about patterns and relationships in data presented in tables, sequences or graphs.

D. Gives appropriate justification of the manipulation of algebraic expressions.

E. Illustrates the concept of a function using concrete models, tables, graphs and symbolic and verbal representations.

F. Uses transformations to illustrate properties of functions and relations and to solve problems.

Competency 005—The teacher understands and uses linear functions to model and solve problems.

The beginning teacher:

A. Demonstrates an understanding of the concept of linear function using concrete models, tables, graphs and symbolic and verbal representations.

B. Demonstrates an understanding of the connections among linear functions, proportions and direct variation.

C. Determines the linear function that best models a set of data.

D. Analyzes the relationship between a linear equation and its graph.

E. Uses linear functions, inequalities and systems to model problems.

F. Uses a variety of representations and methods (e.g., numerical methods, tables, graphs, algebraic techniques) to solve systems of linear equations and inequalities.

G. Demonstrates an understanding of the characteristics of linear models and the advantages and disadvantages of using a linear model in a given situation.

Competency 006—The teacher understands and uses nonlinear functions and relations to model and solve problems.

The beginning teacher:

A. Uses a variety of methods to investigate the roots (real and complex), vertex and symmetry of a quadratic function or relation.

B. Demonstrates an understanding of the connections among geometric, graphic, numeric and symbolic representations of quadratic functions.

C. Analyzes data and represents and solves problems involving exponential growth and decay.

D. Demonstrates an understanding of the connections among proportions, inverse variation and rational functions.

E. Understands the effects of transformations such as $f(x \pm c)$ on the graph of a nonlinear function $f(x)$.

F. Applies properties, graphs and applications of nonlinear functions to analyze, model and solve problems.

G. Uses a variety of representations and methods (e.g., numerical methods, tables, graphs, algebraic techniques) to solve systems of quadratic equations and inequalities.

H. Understands how to use properties, graphs and applications of nonlinear relations including polynomial, rational, radical, absolute value, exponential, logarithmic, trigonometric and piecewise functions and relations to analyze, model and solve problems.

Competency 007—The teacher uses and understands the conceptual foundations of calculus related to topics in middle school mathematics.

The beginning teacher:

A. Relates topics in middle school mathematics to the concept of limit in sequences and series.

B. Relates the concept of average rate of change to the slope of the secant line and instantaneous rate of change to the slope of the tangent line.

C. Relates topics in middle school mathematics to the area under a curve.

D. Demonstrates an understanding of the use of calculus concepts to answer questions about rates of change, areas, volumes and properties of functions and their graphs.

Domain III—Geometry and Measurement

Competency 008—The teacher understands measurement as a process.

The beginning teacher:

A. Selects and uses appropriate units of measurement (e.g., temperature, money, mass, weight, area, capacity, density, percents, speed, acceleration) to quantify, compare and communicate information.

B. Develops, justifies and uses conversions within measurement systems.

C. Applies dimensional analysis to derive units and formulas in a variety of situations (e.g., rates of change of one variable with respect to another) and to find and evaluate solutions to problems.

D. Describes the precision of measurement and the effects of error on measurement.

E. Applies the Pythagorean theorem, proportional reasoning and right triangle trigonometry to solve measurement problems.

Competency 009—The teacher understands the geometric relationships and axiomatic structure of Euclidean geometry.

The beginning teacher:

A. Understands concepts and properties of points, lines, planes, angles, lengths and distances.

B. Analyzes and applies the properties of parallel and perpendicular lines.

C. Uses the properties of congruent triangles to explore geometric relationships and prove theorems.

D. Describes and justifies geometric constructions made using a compass and straight edge and other appropriate technologies.

E. Applies knowledge of the axiomatic structure of Euclidean geometry to justify and prove theorems.

Competency 010—The teacher analyzes the properties of two- and three-dimensional figures.

The beginning teacher:

A. Uses and understands the development of formulas to find lengths, perimeters, areas and volumes of basic geometric figures.

B. Applies relationships among similar figures, scale and proportion and analyzes how changes in scale affect area and volume measurements.

C. Uses a variety of representations (e.g., numeric, verbal, graphic, symbolic) to analyze and solve problems involving two- and three-dimensional figures such as circles, triangles, polygons, cylinders, prisms and spheres.

D. Analyzes the relationship among three-dimensional figures and related two-dimensional representations (e.g., projections, cross-sections, nets) and uses these representations to solve problems.

Competency 011—The teacher understands transformational geometry and relates algebra to geometry and trigonometry using the Cartesian coordinate system.

The beginning teacher:

A. Describes and justifies geometric constructions made using a reflection device and other appropriate technologies.

B. Uses translations, reflections, glide-reflections and rotations to demonstrate congruence and to explore the symmetries of figures.

C. Uses dilations (expansions and contractions) to illustrate similar figures and proportionality.

D. Uses symmetry to describe tessellations and shows how they can be used to illustrate geometric concepts, properties and relationships.

E. Applies concepts and properties of slope, midpoint, parallelism and distance in the coordinate plane to explore properties of geometric figures and solve problems.

F. Applies transformations in the coordinate plane.

G. Uses the unit circle in the coordinate plane to explore properties of trigonometric functions.

Domain IV—Probability and Statistics

Competency 012—The teacher understands how to use graphical and numerical techniques to explore data, characterize patterns and describe departures from patterns.

The beginning teacher:

A. Organizes and displays data in a variety of formats (e.g., tables, frequency distributions, stem-and-leaf plots, box-and-whisker plots, histograms, pie charts).

B. Applies concepts of center, spread, shape and skewness to describe a data distribution.

C. Supports arguments, makes predictions and draws conclusions using summary statistics and graphs to analyze and interpret one-variable data.

D. Demonstrates an understanding of measures of central tendency (e.g., mean, median, mode) and dispersion (e.g., range, interquartile range, variance, standard deviation).

E. Analyzes connections among concepts of center and spread, data clusters and gaps, data outliers and measures of central tendency and dispersion.

F. Calculates and interprets percentiles and quartiles.

Competency 013—The teacher understands the theory of probability.

The beginning teacher:

A. Explores concepts of probability through data collection, experiments and simulations.

B. Uses the concepts and principles of probability to describe the outcome of simple and compound events.

C. Generates, simulates and uses probability models to represent a situation.

D. Determines probabilities by constructing sample spaces to model situations.

E. Solves a variety of probability problems using combinations, permutations and geometric probability (i.e., probability as the ratio of two areas).

F. Uses the binomial, geometric and normal distributions to solve problems.

Competency 014—The teacher understands the relationship among probability theory, sampling and statistical inference and how statistical inference is used in making and evaluating predictions.

The beginning teacher:

A. Applies knowledge of designing, conducting, analyzing and interpreting statistical experiments to investigate real-world problems.

B. Demonstrates an understanding of random samples, sample statistics and the relationship between sample size and confidence intervals.

C. Applies knowledge of the use of probability to make observations and draw conclusions from single variable data and to describe the level of confidence in the conclusion.

D. Makes inferences about a population using binomial, normal and geometric distributions.

E. Demonstrates an understanding of the use of techniques such as scatter plots, regression lines, correlation coefficients and residual analysis to explore bivariate data and to make and evaluate predictions.

Domain V—Mathematical Processes and Perspectives

Competency 015—The teacher understands mathematical reasoning and problem solving.

The beginning teacher:

A. Demonstrates an understanding of proof, including indirect proof, in mathematics.

B. Applies correct mathematical reasoning to derive valid conclusions from a set of premises.

C. Demonstrates an understanding of the use of inductive reasoning to make conjectures and deductive methods to evaluate the validity of conjectures.

D. Applies knowledge of the use of formal and informal reasoning to explore, investigate and justify mathematical ideas.

E. Recognizes that a mathematical problem can be solved in a variety of ways and selects an appropriate strategy for a given problem.

F. Evaluates the reasonableness of a solution to a given problem.

G. Applies content knowledge to develop a mathematical model of a real-world situation and analyzes and evaluates how well the model represents the situation.

H. Demonstrates an understanding of estimation and evaluates its appropriate uses.

Competency 016—The teacher understands mathematical connections within and outside of mathematics and how to communicate mathematical ideas and concepts.

The beginning teacher:

A. Recognizes and uses multiple representations of a mathematical concept (e.g., a point and its coordinates, the area of circle as a quadratic function in *r*, probability as the ratio of two areas).

B. Uses mathematics to model and solve problems in other disciplines, such as art, music, science, social science and business.

C. Expresses mathematical statements using developmentally appropriate language, standard English, mathematical language and symbolic mathematics.

D. Communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphic, pictorial, symbolic, concrete).

E. Demonstrates an understanding of the use of visual media such as graphs, tables, diagrams and animations to communicate mathematical information.

F. Uses the language of mathematics as a precise means of expressing mathematical ideas.

G. Understands the structural properties common to the mathematical disciplines.

H. Explores and applies concepts of financial literacy as it relates to teaching students (e.g., describe the basic purpose of financial institutions, distinguish the difference between gross income and net income, identify various savings options, define different types of taxes, identify the advantages and disadvantages of different methods of payments).

I. Applies mathematics to model and solve problems to manage financial resources effectively for lifetime financial security as it relates to teaching students (e.g., distinguish between fixed and variable expenses, calculate profit in a given situation develop a system for keeping and using financial records, describe actions that might be taken to balance a budget when expenses exceed income and balance a simple budget.)

Domain VI—Mathematical Learning, Instruction and Assessment

Competency 017—The teacher understands how children learn and develop mathematical skills, procedures and concepts.

The beginning teacher:

A. Applies theories and principles of learning mathematics to plan appropriate instructional activities for all students.

B. Understands how students differ in their approaches to learning mathematics with regard to diversity.

C. Uses students' prior mathematical knowledge to build conceptual links to new knowledge and plans instruction that builds on students' strengths and addresses students' needs.

D. Understands how learning may be assisted through the use of mathematics manipulatives and technological tools.

E. Understands how to motivate students and actively engage them in the learning process by using a variety of interesting, challenging and worthwhile mathematical tasks in individual, small-group and large-group settings.

F. Understands how to provide instruction along a continuum from concrete to abstract.

G. Recognizes the implications of current trends and research in mathematics and mathematics education.

Competency 018—The teacher understands how to plan, organize and implement instruction using knowledge of students, subject matter and statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) to teach all students to use mathematics.

The beginning teacher:

A. Demonstrates an understanding of a variety of instructional methods, tools and tasks that promote students' ability to do mathematics described in the TEKS.

B. Understands planning strategies for developing mathematical instruction as a discipline of interconnected concepts and procedures.

C. Develops clear learning goals to plan, deliver, assess and reevaluate instruction based on the TEKS.

D. Understands procedures for developing instruction that establishes transitions between concrete, symbolic and abstract representations of mathematical knowledge.

E. Applies knowledge of a variety of instructional delivery methods, such as individual, structured small-group and large-group formats.

F. Understands how to create a learning environment that provides all students, including English-language learners, with opportunities to develop and improve mathematical skills and procedures.

G. Demonstrates an understanding of a variety of questioning strategies to encourage mathematical discourse and to help students analyze and evaluate their mathematical thinking.

H. Understands how technological tools and manipulatives can be used appropriately to assist students in developing, comprehending and applying mathematical concepts.

I. Understands how to relate mathematics to students' lives and a variety of careers and professions.

Competency 019—The teacher understands assessment and uses a variety of formal and informal assessment techniques to monitor and guide mathematics instruction and to evaluate student progress.

The beginning teacher:

A. Demonstrates an understanding of the purpose, characteristics and uses of various assessments in mathematics, including formative and summative assessments.

B. Understands how to select and develop assessments that are consistent with what is taught and how it is taught.

C. Demonstrates an understanding of how to develop a variety of assessments and scoring procedures consisting of worthwhile tasks that assess mathematical understanding, common misconceptions and error patterns.

D. Understands how to evaluate a variety of assessment methods and materials for reliability, validity, absence of bias, clarity of language and appropriateness of mathematical level.

E. Understands the relationship between assessment and instruction and knows how to evaluate assessment results to design, monitor and modify instruction to improve mathematical learning for all students, including English-language learners.

This reference material will also be available to you during the exam. To access it, click on the Reference Materials icon located in the lower-left corner of the screen.

Definitions and Formulas

CALCULUS

First Derivative: $f'(x) = \dfrac{dy}{dx}$

Second Derivative: $f''(x) = \dfrac{d^2y}{dx^2}$

PROBABILITY

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$

ALGEBRA

i — $i^2 = -1$

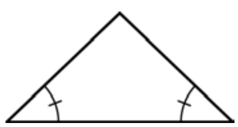
$\boldsymbol{A}^{-1}$ — inverse of matrix A

$\boldsymbol{A = P\left(1 + \frac{r}{n}\right)^{nt}}$ — Compound interest,
where A is the final value
P is the principal
r is the interest rate
t is the term
n is the number of divisions within the term

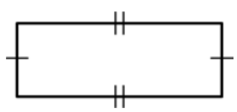
$[\boldsymbol{x}] = \boldsymbol{n}$ — Greatest integer function,
where n is the integer such that $n \le x < n + 1$

GEOMETRY

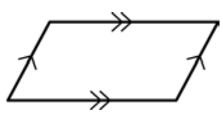
Congruent Angles



Congruent Sides



Parallel Sides



Circumference of a Circle

$C = 2\pi r$

VOLUME

Cylinder: (area of base) $\times$ height

Cone: $\frac{1}{3}$(area of base) $\times$ height

Sphere: $\frac{4}{3}\pi r^3$

Prism: (area of base) $\times$ height

AREA

Triangle: $\frac{1}{2}(\text{base} \times \text{height})$

Rhombus: $\frac{1}{2}(\text{diagonal}_1 \times \text{diagonal}_2)$

Trapezoid: $\frac{1}{2}$ height $(\text{base}_1 + \text{base}_2)$

Sphere: $4\pi r^2$

Circle: πr^2

Lateral surface area of cylinder: $2\pi rh$

TRIGONOMETRY

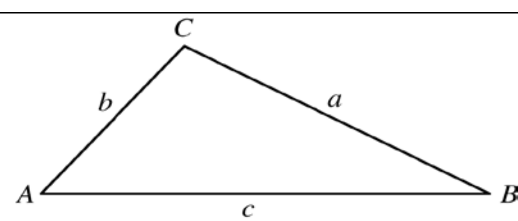
Law of Sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Law of Cosines:

$c^2 = a^2 + b^2 - 2ab\cos C$

$b^2 = a^2 + c^2 - 2ac\cos B$

$a^2 = b^2 + c^2 - 2bc\cos A$



End of Definitions and Formulas

Section 4: Sample Selected-Response Questions Mathematics 4–8 (115)

This section presents some sample exam questions for you to review as part of your preparation for the exam. To demonstrate how each competency may be assessed, sample questions are accompanied by the competency that they measure. While studying, you may wish to read the competency before and after you consider each sample question. Please note that the competency statements do not appear on the actual exam.

For each sample exam question, there is a correct answer and a rationale for each answer option. The sample questions are included to illustrate the formats and types of questions you will see on the exam; however, your performance on the sample questions should not be viewed as a predictor of your performance on the actual exam.

The following reference materials will be available to you during the exam:

- Definitions and Formulas (see page 12)

Domain I—Number Concepts

Competency 001—The teacher understands the structure of number systems, the development of a sense of quantity and the relationship between quantity and symbolic representations.

1. Which of the following right triangles has a hypotenuse with a length that is an irrational number?

   A. A right triangle with leg lengths of 4 and 3

   B. A right triangle with leg lengths of 12 and 5

   C. A right triangle with leg lengths of 24 and 7

   D. A right triangle with leg lengths of 25 and 9

   Answer _____

Competency 002—The teacher understands number operations and computational algorithms.

2. Rectangle I has dimensions a and b, and rectangle II has dimensions $a - 2$ and $b + 2$, where $a > 2$ and $b > 0$. Which of the following must be true?

   A. The area of rectangle I is less than the area of rectangle II.

   B. The area of rectangle I is greater than the area of rectangle II.

   C. The perimeter of rectangle I is less than the perimeter of rectangle II.

   D. The perimeter of rectangle I is equal to the perimeter of rectangle II.

   Answer _____

3. Which of the following is equivalent to the product $(3 + 2i)(4 + 3i)$?

A. $6 + 17i$

B. $12 + 6i$

C. $18 + 17i$

D. $12 + 17i$

Answer _____

4. Which of the following is equivalent to $2\sqrt{3}(\sqrt{2} + \sqrt{3})$?

A. $2\sqrt{15}$

B. $4\sqrt{15}$

C. $2\sqrt{6} + 6$

D. 18

Answer _____

Competency 003—The teacher understands ideas of number theory and uses numbers to model and solve problems within and outside of mathematics.

5. A traveler in Europe noticed on a certain day that 3.85 euros was worth 5.00 United States dollars. Based on this rate of exchange, 10 euros is approximately equal to how many United States dollars?

A. 7.70

B. 9.25

C. 10.77

D. 12.99

Answer _____

Domain II—Patterns and Algebra

Competency 004—The teacher understands and uses mathematical reasoning to identify, extend and analyze patterns and understands the relationships among variables, expressions, equations, inequalities, relations and functions.

6. An amount of 10 gallons of water is stored in a 15-gallon container. During the first 4 hours, the water evaporates from the container at a rate of 0.1 gallon per hour. During the next 5 hours, the water evaporates from the container at a rate of 0.3 gallon per hour. Which of the following functions represents the volume of water in the container, at time t, where $0 \le t \le 9$?

A. $f(t) = 10 - 0.4t \quad 0 \le t \le 9$

B. $f(t) = 9.6 - 0.3t \quad 0 \le t \le 9$

C. $f(t) = \begin{cases} 10 - 0.1t & 0 \le t \le 4 \\ 9.6 - 0.3t & 4 < t \le 9 \end{cases}$

D. $f(t) = \begin{cases} 15 - 0.1t & 0 \le t \le 4 \\ 10 - 0.4t & 4 < t \le 9 \end{cases}$

Answer _____

7. Each week last year, a small manufacturer earned a profit by selling handbags. The weekly profit P from selling x handbags is modeled by the function $P(x) = -0.5x^2 + 40x - 300$. Based on the model, what was the maximum weekly profit, in dollars, that the manufacturer could have earned last year?

A. $300

B. $450

C. $500

D. $700

Answer _____

8. Ms. Johnston is a sales associate at a jewelry store. Her total weekly earnings consist of a wage of $10 per hour plus a 10 percent commission on her total sales for the week. One week Ms. Johnston worked 30 hours and had total sales of x dollars. Which of the following represents her total weekly earnings y, in dollars, for that week?

A. $y = 10x + 0.01$

B. $y = 0.1x + 300$

C. $y = 0.1(x + 300)$

D. $y = 30(0.1x + 10)$

Answer _____

Competency 005—The teacher understands and uses linear functions to model and solve problems.

9. Which of the following is the equation of the line in the xy-plane that passes through the points (–7, –2) and (–2, –7)?

A. $x + y = -9$

B. $x - y = -9$

C. $x - y = -5$

D. $-x + y = -5$

Answer _____

10. In the xy-plane, line segment AB is bisected by line segment CD, and the coordinates of the point of intersection are (–2, –3). If the coordinates of A are (–8, –1), what are the coordinates of point B ?

A. (5, –5)

B. (4, –5)

C. (–5, –2)

D. (8, 1)

Answer _____

11. A restaurant charges $9.00 for a large pizza and $1.75 for each topping selected. Let C be the total cost of a pizza and let t be the number of toppings selected. Which of the following is an equation for the total cost C of a pizza?

A. $C = 9.00 + 1.75t$

B. $C = (1.75 + 9.00)t$

C. $C = 9.00t + 1.75$

D. $C = (1.75)(9.00)t$

Answer _____

Competency 006—The teacher understands and uses nonlinear functions and relations to model and solve problems.

12. Which of the following points is the vertex of the graph of $y = 2x^2 - 8x + 1$ in the xy-plane?

A. (2,13)

B. (0,1)

C. (4,1)

D. (2,–7)

Answer _____

13. Which of the following values of x satisfies $2x^2 + 5x - 3 < 0$?

A. –3

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. 2

Answer _____

14. An eighth-grade mathematics teacher is preparing a lesson about exponential decay and plans to use an example involving half-life. The teacher explains that half-life is the amount of time required for the initial quantity of a substance to reduce by half, and the teacher then gives an example of a radioactive substance with a half-life of 40 years. If the initial quantity of the radioactive substance is P grams, what is the quantity, in grams, that will remain after 200 years?

A. $\left(\frac{1}{32}\right)P$

B. $\left(\frac{1}{16}\right)P$

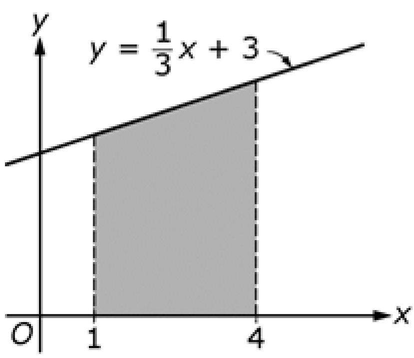
C. $\left(\frac{1}{5}\right)P$

D. $5P$

Answer _____

Competency 007—The teacher uses and understands the conceptual foundations of calculus related to topics in middle school mathematics.

Use the figure below to answer the question that follows.



15. A geometry teacher developed a lesson that incorporates solving linear equations using algebra and finding the area of geometric shapes using geometry. Which of the following calculus topics could be demonstrated by finding the area of the trapezoid above?

A. The derivative of a function at a point

B. The definite integral

C. The limit of a function of *x* as *x* goes to infinity

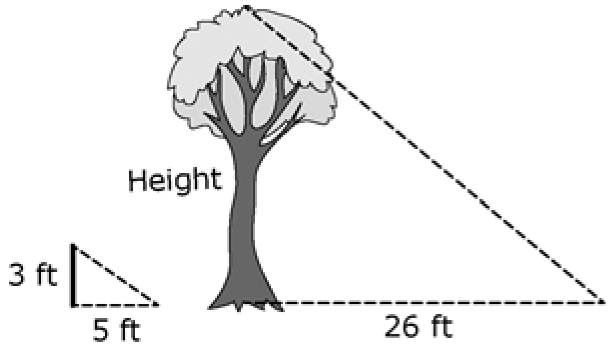
D. Newton's method to find the zeros of a function

Answer _____

Domain III—Geometry and Measurement

Competency 008—The teacher understands measurement as a process.

Use the figure below to answer the question that follows.



Note: Figure not drawn to scale.

16. At a certain time of day, a student measured the height of the shadow of a yardstick, held vertically, to be 5 feet. At the same time of day, the student measured the length of the shadow of the tree to be 26 feet. To the nearest foot, what is the height of the tree?

A. 16 feet

B. 24 feet

C. 30 feet

D. 43 feet

Answer _____

17. Frank completed a 400-meter race in 75 seconds. Which of the following is closest to Frank's speed in kilometers per hour?

A. 17 kilometers per hour

B. 19 kilometers per hour

C. 21 kilometers per hour

D. 23 kilometers per hour

Answer _____

18. Tony purchased 100 kilograms of decorative stones. Each stone weighs approximately 10 grams. Which of the following is the best estimate of the number of stones purchased?

A. 10^3

B. 10^4

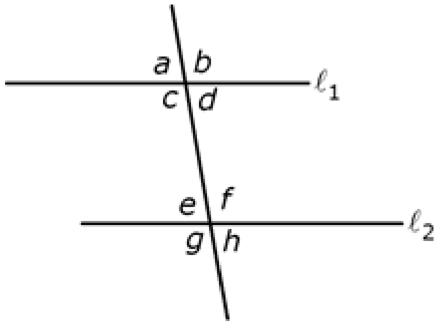
C. 10^5

D. 10^6

Answer _____

Competency 009—The teacher understands the geometric relationships and axiomatic structure of Euclidean geometry.

Use the figure below to answer the question that follows.



19. In the diagram above, ℓ_1 is parallel to ℓ_2. If the measure of angle b is 100°, what is the measure of angle e?

A. 100°

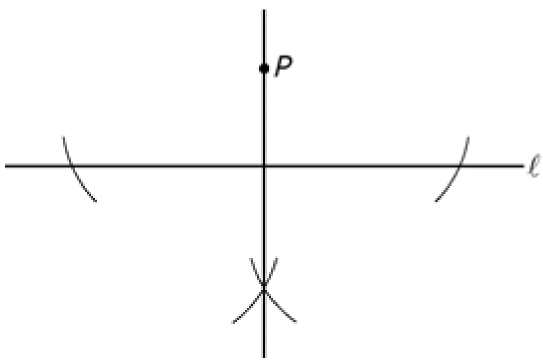
B. 95°

C. 80°

D. 75°

Answer _____

Use the figure below to answer the question that follows.



20. Which of the following describes the geometric construction above, where the construction uses only a compass and a straightedge?

A. The locus of points that are equidistant from line ℓ and point *P*

B. The line perpendicular to line ℓ and passing through point *P*

C. The extension of line ℓ

D. The line parallel to line ℓ and passing through point *P*

Answer _____

21. Let *ABC* be a triangle, where *AB* has length 4 and *BC* has length 8. For which of the following possible lengths of *AC* is *ABC* an obtuse triangle?

Select _all_ that apply.

A. 6

B. 7

C. 8

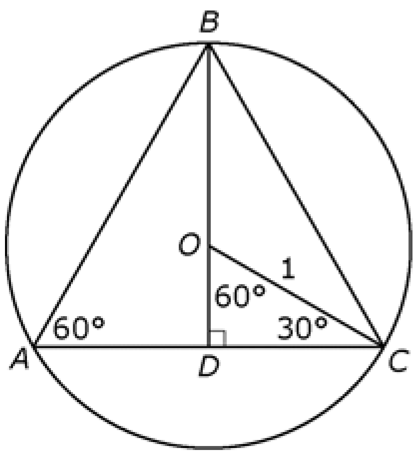
D. 9

E. 10

Answer _____

Competency 010—The teacher analyzes the properties of two- and three-dimensional figures.

Use the figure below to answer the question that follows.



22. Equilateral triangle *ABC* is inscribed in a circle with center *O* and a radius of 1, as shown above. The height of the triangle is *BD*. What is the area of triangle *ABC*?

A. $\frac{\sqrt{3}}{2}$

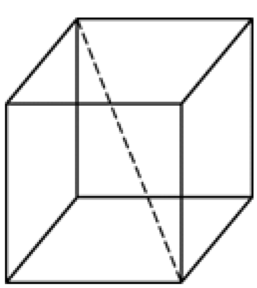
B. $\frac{\sqrt{3}}{8}$

C. $\frac{3\sqrt{3}}{2}$

D. $\frac{3\sqrt{3}}{4}$

Answer _____

Use the cube below to answer the question that follows.



23. In the cube shown above, a student measured the length of a diagonal to be 4.5 centimeters. Which of the following is the best estimate of the volume of the cube?

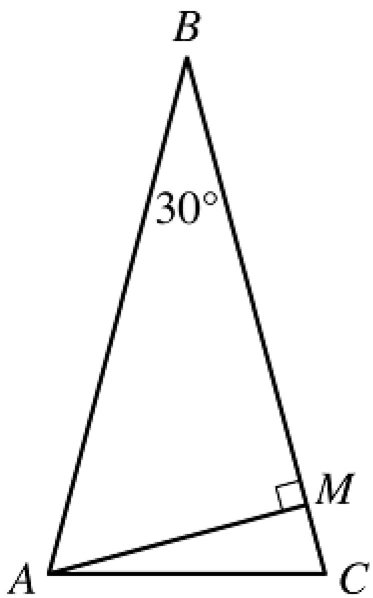
A. 121.5 cubic centimeters

B. 91.1 cubic centimeters

C. 17.5 cubic centimeters

D. 2.6 cubic centimeters

Answer _____



24. In triangle *ABC* shown, *AB* = *BC*, the measure of angle *ABC* is 30 degrees, and line segment *AM* is perpendicular to side *BC*. What is the degree measure of angle *MAC*?

A. 10°

B. 15°

C. 60°

D. 75°

Answer _____

25. A circle with radius *r* has a circumference of 48. What is the value of *r*?

A. $r = \frac{24}{\pi}$

B. $r = \sqrt{\frac{48}{\pi}}$

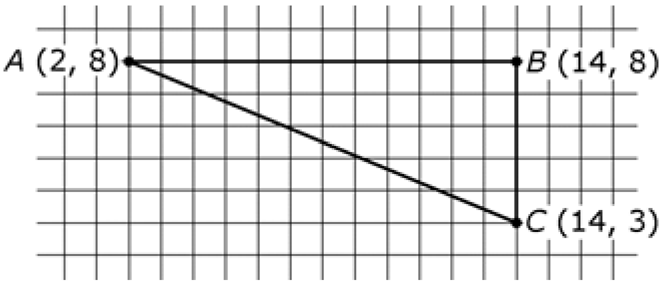
C. $r = \sqrt[3]{\frac{36}{\pi}}$

D. $r = \frac{48}{\pi}$

Answer _____

Competency 011—The teacher understands transformational geometry and relates algebra to geometry and trigonometry using the Cartesian coordinate system.

Use the figure below to answer the question that follows.



26. Triangle *A′B′C′* has a hypotenuse of length 52 and is a dilation of triangle *ABC* shown above. What is the scale factor used to dilate triangle *ABC* to transform it to triangle *A′B′C′*?

A. $\frac{1}{4}$

B. $\frac{4}{13}$

C. $\frac{13}{4}$

D. 4

Answer _____

Domain IV—Probability and Statistics

Competency 012—The teacher understands how to use graphical and numerical techniques to explore data, characterize patterns and describe departures from patterns.

Use the list below to answer the question that follows.

90, 70, 60, 75, 80, 82, 85, 88, 80, *x*

27. The list above shows ten scores from a recent test in a math class. The range of the ten scores is 30, and the interquartile range is 10. Which of the following could be the value of *x* ?

A. 74

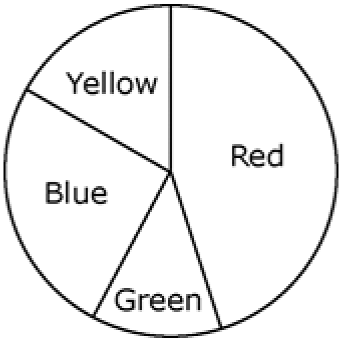
B. 84

C. 86

D. 96

Answer _____

Use the circle graph below to answer the question that follows.



28. Ms. Jefferson read an article to her class describing a survey of students who were asked to choose their favorite color among the colors yellow, blue, green, and red. The graph above shows the results of the survey. Ms. Jefferson tells the class that the survey results are representative of all students, and she asks the class to predict how many of the 850 students in their school would choose either yellow or green. Which of the following is the best estimate?

A. 20

B. 25

C. 150

D. 250

Answer _____

29. The height of each student at Jefferson Middle School was measured and recorded. Joseph was told that his height was at the 60^{th} percentile of the heights of the students. Which of the following must be true?

A. The heights of 4 students are greater than Joseph's height.

B. Joseph's height is 60 inches.

C. The heights of at least 60 percent of the students are less than or equal to Joseph's height.

D. If the height of the tallest student is 70 inches, then Joseph's height is (0.6)(70) = 48 inches.

Answer _____

List *L*: 4, 9, 4, 8, 9, 8, 11, 10, 9

30. List *L* is shown above. Let $\bar{x}$ be the mean, let m be the median, and let D be the mode of the numbers in list *L*. Which of the following is true?

A. $\bar{x} = m$ and $m = D$

B. $\bar{x} = m$ and $m < D$

C. $\bar{x} < m$ and $m = D$

D. $\bar{x} < m$ and $m < D$

Answer _____

Competency 013—The teacher understands the theory of probability.

31. To form a committee, a principal will choose 3 students from a group of 5 seventh-grade students and 2 students from a group of 6 eighth-grade students. What is the total number of different committees the principal could select?

A. 16

B. 150

C. 180

D. 720

Answer _____

32. Six swimmers are competing in a 25-meter race. If there are no ties, how many different combinations are possible for a first-, second-, and third-place finish?

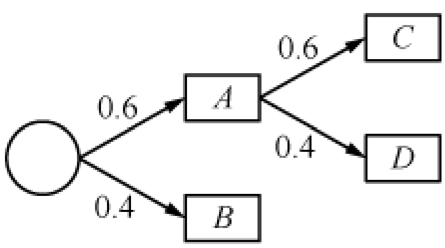
A. 216

B. 120

C. 18

D. 15

Answer _____



33. Mr. Garcia showed his mathematics class the probability tree diagram shown above, where 0.6 and 0.4 represent the probability that an event will occur. Based on the diagram, what is the probability that event *D* will occur?

A. 1.00

B. 0.36

C. 0.24

D. 0.20

Answer _____

34. A high school issues each new student a 7-character identifier. The first 3 characters are the first 3 letters in the student's last name, all capital letters from the 26-character English alphabet. The 3 letters are followed by 4 randomly generated integers from 0 to 9, inclusive. Repetition of letters and integers is allowed. How many unique identifiers can be generated?

A. $(26^3)(10^4)$

B. $(26)(25)(24)(10^4)$

C. $(26)(25)(24)(10)(9)(8)(7)$

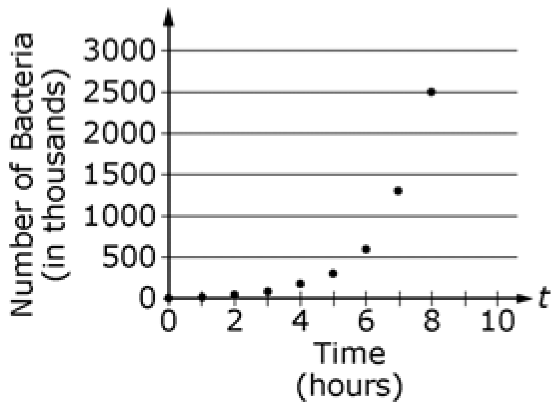
D. $\frac{(26!)(10!)}{(3!)(23!)(4!)(6!)}$

Answer _____

Competency 014—The teacher understands the relationship among probability theory, sampling and statistical inference and how statistical inference is used in making and evaluating predictions.

Use the information below to answer the question that follows.

Time *t* (hours)	Number of Bacteria (in thousands)
0	10
1	18
2	35
3	85
4	170
5	300
6	600
7	1300
8	2500



35. Every hour, a scientist counted the number of bacteria growing in a certain medium. The scientist recorded the results in a table and produced the scatterplot shown above. If $P(t)$ is a mathematical model for the number of bacteria, in thousands, at time t hours, which of the following expressions is the best fit for $P(t)$?

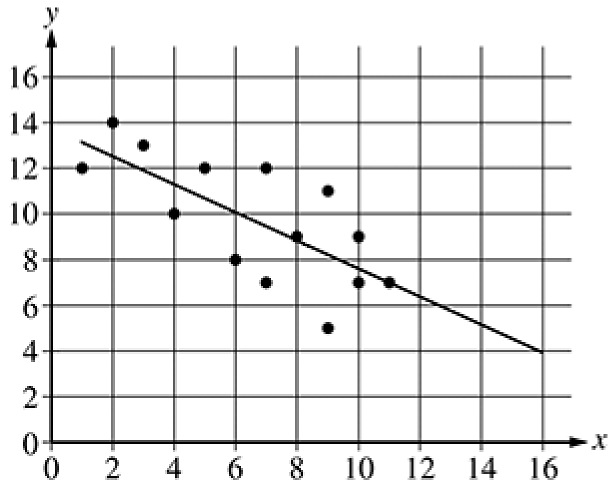
A. $P(t) = 300t + 10$

B. $P(t) = 300t^2 + 100t + 10$

C. $P(t) = 10(2^t)$

D. $P(t) = 10(\ln(2t + 1))$

Answer _____



36. The scatterplot shows the relationship between two sets of data, x and y. The line of best fit of the relationship between the two data sets is shown. Based on the line of best fit, which of the following is the best estimate of the value of y when the value of x is 14?

A. 0

B. 2

C. 5

D. 7

Answer _____

37. In 2016 there were 324,582 licensed teachers in a certain state. The ages of the teachers were modeled with a normal distribution. The mean age was 43.5 years old, with a standard deviation of 8.2 years. Based on this distribution, which of the following is the best estimate of the number of licensed teachers in the state in 2016 with an age of 59.9 years old or greater?

A. 8000 teachers

B. 16,000 teachers

C. 55,000 teachers

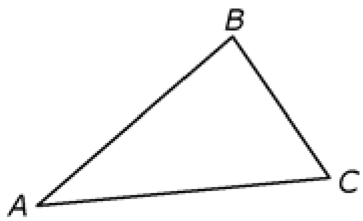
D. 110,000 teachers

Answer _____

Domain V—Mathematical Processes and Perspectives

Competency 015—The teacher understands mathematical reasoning and problem solving.

Use the figure and the proof below to answer the question that follows.



Given: In triangle *ABC* shown, *AB* > *BC*

Prove: ∠*A* ≠ ∠*C*, that is the measure of angle *A* is not equal to the measure of angle *C*.

Proof by contradiction: Assume that ∠*A* ≅ ∠*C*.

If ∠*A* ≅ ∠*C*, then $\overline{AB} \cong \overline{AC}$ by the ______________. However, this theorem contradicts the given information that *AB* > *BC*. Therefore, the assumption that ∠*A* ≅ ∠*C* must be false, and so the measures of the angles cannot be equal and ∠*A* ≠ ∠*C*.

38. In the proof above, which of the following theorems correctly fills in the blank?

A. corresponding angles theorem

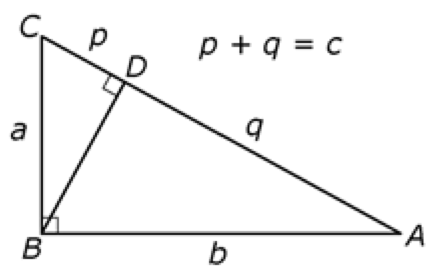
B. angle bisector theorem

C. isosceles triangle theorem

D. congruence of triangles with the side-side-side property

Answer _____

Use the figure and the proof below to answer the question that follows.



Statement	Reason
$\frac{p}{a} = \frac{a}{c}$ and $\frac{q}{b} = \frac{b}{c}$	1. ?
$p = \frac{a^2}{c}$ and $q = \frac{b^2}{c}$	2. Multiplication property of equality
$c = p + q = \frac{a^2}{c} + \frac{b^2}{c}$	3. Substitution
$c^2 = a^2 + b^2$	4. Multiplication property of equality

39. A seventh-grade mathematics teacher presented the proof of the Pythagorean theorem shown above, with one missing reason for students to supply. What is the missing reason?

A. Triangles *CDB* and *BDA* are similar.

B. Apply the angle-side-angle theorem to triangle *BDA*.

C. Apply the side-angle-side theorem to triangle *CBA*.

D. Triangles *CBA* and *CDB* are similar, and triangles *CBA* and *BDA* are similar.

Answer _____

Competency 016—The teacher understands mathematical connections within and outside of mathematics and how to communicate mathematical ideas and concepts.

40. A teacher would like to instruct students about semiregular tessellations of a plane. A semiregular tessellation of a plane uses more than one type of regular polygon. Also, every vertex in the tessellation has the same arrangement of polygons around it, where the sum of the angles around the vertex is 360°. The teacher proposes a tessellation that uses three types of regular polygons: a 15-gon, a triangle, and a third type. What is the third type of regular polygon?

A. A pentagon

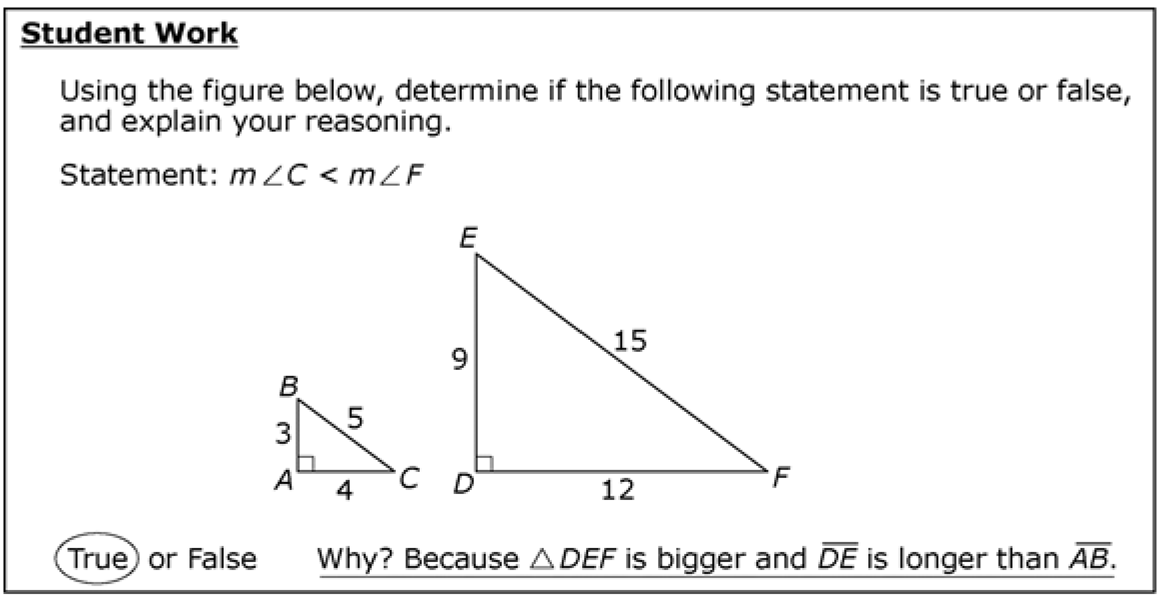
B. A hexagon

C. An octagon

D. A decagon

Answer _____

Use the student work below to answer the question that follows.



41. Which of the following is the most appropriate way for a mathematics teacher to respond to the student work shown?

A. The student's work is correct.

B. The student's calculations are correct, but the teacher should ask the student to calculate the measure of each angle.

C. The student's work is incorrect because the angle measures are not given and cannot be determined from the information provided.

D. The student's work is incorrect because the triangles are similar; therefore, $m\angle C = m\angle F$.

Answer _____

42. Rachel and Peter deposited $25,000 into a new savings account with an annual interest rate of 3.5 percent, compounded semiannually. If they do not make any withdrawals from the account or make any other deposits to the account, which of the following is closest to the balance in the account after three years?

A. $26,335.60

B. $27,625.00

C. $27,742.56

D. $30,731.38

Answer _____

43. In 2016 Mr. Schuppan paid an annual property tax of $1500. In 2017 Mr. Schuppan paid an annual property tax of $1650. What was the percent increase of Mr. Schuppan's property tax from 2016 to 2017?

A. 8.03%

B. 9.09%

C. 10.0%

D. 15.0%

Answer _____

Domain VI—Mathematical Learning, Instruction and Assessment

Competency 017—The teacher understands how children learn and develop mathematical skills, procedures and concepts.

Use the example below to answer the question that follows.

$$\frac{2x^4 - 3x^3 + x^2 - 4x + 5}{x - 1}$$

44. A teacher is preparing a unit on polynomial long division and would like to use the problem shown above as an example. Before discussing the example, of the following, which concept is best for the teacher to review?

A. Multiplying two rational expressions

B. Vertical asymptotes

C. The additive property of equality

D. The division algorithm for real numbers

Answer _____

$$\frac{36x^3y^{-2}z^4}{18x^2y^{-3}z^6}$$

45. A teacher is preparing a unit about simplifying algebraic expressions and would like to use the expression shown above as an example. Before discussing the example, which of the following concepts is best for the teacher to review?

A. The zero-product property of multiplication

B. Extraneous solutions

C. The laws of exponents

D. The properties of multiplication of real numbers

Answer _____

Competency 018—The teacher understands how to plan, organize and implement instruction using knowledge of students, subject matter and statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) to teach all students to use mathematics.

46. Marcus is renting a bicycle. The rental requires a down payment of $15 plus $6 for each hour the bicycle is rented. If C is the total charge and t is the number of hours that Marcus rented the bicycle, which of the following equations represents the relationship between the amount of time he rented the bicycle and the total cost?

A. $C = \frac{1}{6}t + 15$

B. $C = 6t + 15$

C. $C = 15t + 6$

D. $C = 15(t - 1) + 6$

Answer _____

47. A mathematics teacher assigns students in a sixth-grade class to keep a mathematics diary. Every day, the students are asked to record when and how they use mathematics in their daily lives. At the end of each week, the students each write a short report on their use of mathematics, and the teacher reviews their diaries. Which of the following is the teacher demonstrating with this activity?

A. An understanding of a variety of questioning strategies to encourage mathematical discourse and to help students analyze, evaluate, and communicate their mathematical thinking

B. An understanding of the use of inductive reasoning to make conjectures and deductive methods to evaluate the validity of conjectures

C. An understanding of the purpose, characteristics, and uses of summative assessments

D. An understanding of how technological tools and manipulatives can be used appropriately to assist students in developing, comprehending, and applying mathematical concepts

Answer _____

48. Let $f(x) = (x - 100)(x - 125)(x - 150)(x - 175)$. A student wants to use a graphing calculator to find the points in the *xy*-plane where the graph of *f* crosses the *x*-axis. Which of the following bounds on *x* can be used in a viewing window so that the calculator displays all the points where the graph of *f* crosses the *x*-axis?

A. $-391{,}000 \leq x \leq 75$

B. $-200 \leq x \leq -75$

C. $-100 \leq x \leq 99$

D. $75 \leq x \leq 200$

Answer _____

Competency 019—The teacher understands assessment and uses a variety of formal and informal assessment techniques to monitor and guide mathematics instruction and to evaluate student progress.

49. A mathematics teacher finished a unit on adding fractions and gave a formative assessment. While grading the assessment, the teacher found that more than half of the students were making the error $\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$. Which of the following is the best way for the teacher to address this error?

A. Reviewing fraction addition and giving another formative assessment with questions designed to determine common errors among the students

B. Moving to the next section in the text and including questions on fraction addition on the summative review at the end of the semester

C. Asking the students to perform a search of newspapers and Web sites and document how fractions are used in news stories

D. Moving to the next section in the text and assigning extra-credit homework on fraction addition

Answer _____

50. A mathematics teacher finishes a unit on multiplying expressions with exponents and gives a formative assessment. While grading the assessment, the teacher finds that more than half of the students made the error $(2^4)(2^5) = 2^{20}$. Which of the following is the best description of the error?

A. The students multiplied the exponents instead of adding the exponents.

B. The students should have expanded $2^4 = 16$ and $2^5 = 32$ then multiplied the numbers instead of using the properties of exponents.

C. The students should not have omitted the multiplication sign between the two numbers in parentheses.

D. The students should have expanded the answer, 2^{20}, to an integer.

Answer _____

Section 4: Sample Selected-Response Answers and Rationales
Mathematics 4–8 (115)

This section presents some sample exam questions for you to review as part of your preparation for the exam. To demonstrate how each competency may be assessed, sample questions are accompanied by the competency that they measure. While studying, you may wish to read the competency before and after you consider each sample question. Please note that the competency statements do not appear on the actual exam.

For each sample exam question, there is a correct answer and a rationale for each answer option. The sample questions are included to illustrate the formats and types of questions you will see on the exam; however, your performance on the sample questions should not be viewed as a predictor of your performance on the actual exam.

The following reference materials will be available to you during the exam:

- Definitions and Formulas (see page 12)

Domain I—Number Concepts

Competency 001—The teacher understands the structure of number systems, the development of a sense of quantity and the relationship between quantity and symbolic representations.

1. Which of the following right triangles has a hypotenuse with a length that is an irrational number?

   A. A right triangle with leg lengths of 4 and 3

   B. A right triangle with leg lengths of 12 and 5

   C. A right triangle with leg lengths of 24 and 7

   D. A right triangle with leg lengths of 25 and 9

Answer

Option D is correct because the length of the hypotenuse of a right triangle is given by the square root of the sum of the squares of the legs, and $\sqrt{25^2 + 9^2} = \sqrt{625 + 81} = \sqrt{706}$, which is an irrational number. **Option A is incorrect** because $\sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$, which is a rational number. **Option B is incorrect** because $\sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$, which is a rational number. **Option C is incorrect** because $\sqrt{24^2 + 7^2} = \sqrt{576 + 49} = \sqrt{625} = 25$, which is a rational number.

Competency 002—The teacher understands number operations and computational algorithms.

2. Rectangle I has dimensions a and b, and rectangle II has dimensions $a - 2$ and $b + 2$, where $a > 2$ and $b > 0$. Which of the following must be true?

A. The area of rectangle I is less than the area of rectangle II.

B. The area of rectangle I is greater than the area of rectangle II.

C. The perimeter of rectangle I is less than the perimeter of rectangle II.

D. The perimeter of rectangle I is equal to the perimeter of rectangle II.

Answer

Option D is correct because the perimeters of rectangles I and II are $2a + 2b$ and $2(a - 2) + 2(b + 2) = 2a - 4 + 2b + 4 = 2a = 2b$, respectively, which are equal. **Option A is incorrect** because in the case $a - b < 2$ the area of rectangle II is $(a - 2)(b + 2) = ab + 2a - 2b - 4 = ab + 2(a - b) - 4$, which is less than $ab + (2)(2) - 4 + ab$, that is, less than the area of rectangle I. **Option B is incorrect** because in the case of $a - b > 2$, the area of rectangle II is $(a - 2)(b + 2) = ab + 2a - 2b - 4 = ab + 2(a - b) - 4$, which is greater than $ab + (2)(2) - 4$ that is, greater than the area of rectangle I. Note that if $a - b = 2$, then the area of rectangles I and II are equal. **Option C is incorrect** because it contradicts option D, which is true.

3. Which of the following is equivalent to the product $(3 + 2i)(4 + 3i)$?

A. $6 + 17i$

B. $12 + 6i$

C. $18 + 17i$

D. $12 + 17i$

Answer

Option A is correct because the product is $12 + 8i + 9i + 6i^2 = 12 + 17i - 6 = 6 + 17i$. **Option B is incorrect** because the products of two binomials were not distributed properly during multiplication. **Option C is incorrect** because i^2 is equivalent to –1, not 1. **Option D is incorrect** because the term $6i^2$ is neglected.

4. Which of the following is equivalent to $2\sqrt{3}(\sqrt{2} + \sqrt{3})$?

A. $2\sqrt{15}$

B. $4\sqrt{15}$

C. $2\sqrt{6} + 6$

D. 18

Answer

Option C is correct because the expression can be simplified using the distributive property: $2\sqrt{3}(\sqrt{2} + \sqrt{3}) = 2(\sqrt{3})(\sqrt{2}) + (2\sqrt{3})(\sqrt{3}) = 2(\sqrt{3})(\sqrt{2}) + 2(\sqrt{3})(\sqrt{3}) = 2\sqrt{6} + 2\sqrt{9} = 2\sqrt{6} + 2(3) = 2\sqrt{6} + 6$. **Option A is incorrect** because $\sqrt{2} + \sqrt{3} \neq \sqrt{5}$. **Option B is incorrect** because $2\sqrt{6} + 2\sqrt{9} \neq 4\sqrt{15}$. **Option D is incorrect** because $2(\sqrt{3})(\sqrt{2}) + 2(\sqrt{3})(\sqrt{3}) \neq 2(6) + 2(3) = 18$.

Competency 003—The teacher understands ideas of number theory and uses numbers to model and solve problems within and outside of mathematics.

5. A traveler in Europe noticed on a certain day that 3.85 euros was worth 5.00 United States dollars. Based on this rate of exchange, 10 euros is approximately equal to how many United States dollars?

A. 7.70

B. 9.25

C. 10.77

D. 12.99

Answer

Option D is correct because the given numbers of dollars, d, and euros, e, can be related by the equation $d = ke$, where $k = \frac{5}{3.85} \approx 1.299$ is the rate of exchange in dollars per euro. Using the rate k when $e = 10$ yields $d \approx$ \$12.99. **Option A is incorrect** because this response results from using an incorrect rate of exchange, $\frac{3.85}{5} =$ 0.77 euro per dollar, misapplied to 10 euros. **Option B is incorrect** because this response corresponds to multiplying d and e and then subtracting 10: $(5)(3.85) - 10$. **Option C is incorrect** because this response corresponds to dividing e by d and then adding 10: $\frac{3.85}{5} + 10$.

Domain II—Patterns and Algebra

Competency 004—The teacher understands and uses mathematical reasoning to identify, extend and analyze patterns and understands the relationships among variables, expressions, equations, inequalities, relations and functions.

6. An amount of 10 gallons of water is stored in a 15-gallon container. During the first 4 hours, the water evaporates from the container at a rate of 0.1 gallon per hour. During the next 5 hours, the water evaporates from the container at a rate of 0.3 gallon per hour. Which of the following functions represents the volume of water in the container, at time t, where $0 \le t \le 9$?

A. $f(t) = 10 - 0.4t \quad 0 \le t \le 9$

B. $f(t) = 9.6 - 0.3t \quad 0 \le t \le 9$

C. $f(t) = \begin{cases} 10 - 0.1t & 0 \le t \le 4 \\ 9.6 - 0.3t & 4 < t \le 9 \end{cases}$

D. $f(t) = \begin{cases} 15 - 0.1t & 0 \le t \le 4 \\ 10 - 0.4t & 4 < t \le 9 \end{cases}$

Answer

Option C is correct because water evaporates at a rate of 0.1 gallon per hour during the first 4 hours. After 4 hours, there are $10 - (0.1)(4) = 9.6$ gallons left. For the next 5 hours, the rate of evaporation is 0.3 gallon per hour. **Option A is incorrect** because the rate of evaporation is not a constant 0.4 gallon per hour. **Option B is incorrect** because the initial amount is 10 gallons of water, and the rate of evaporation is not a constant 0.3 gallon per hour. **Option D is incorrect** for several reasons. For example, the initial amount is 10 gallons of water, not 15.

7. Each week last year, a small manufacturer earned a profit by selling handbags. The weekly profit P from selling x handbags is modeled by the function $P(x) = -0.5x^2 + 40x - 300$. Based on the model, what was the maximum weekly profit, in dollars, that the manufacturer could have earned last year?

A. $300

B. $450

C. $500

D. $700

Answer

Option C is correct because the given function is quadratic, and therefore, its graph is a parabola that opens downward. The maximum possible weekly profit is the value of the function at the vertex of the parabola. The x-coordinate of the vertex can be found using the formula $x = -\frac{b}{2a} = -\frac{40}{2(-0.5)} = 40$, where a and b are the coefficients of the x^2 term and the x term, respectively. Substituting $x = 40$ in the function gives the value $P(40) = 500$ at the vertex. Therefore, the maximum possible profit is $500. **Option A is incorrect** because it corresponds to an x-coordinate of $x = \frac{b}{2} = \frac{40}{2} = 20$ at the vertex, whereby the value would be $P(20) = 300$. **Option B is incorrect** because it corresponds to an x-coordinate of $x = 30$ at the vertex, whereby the value would be $P(30) = 450$. **Option D is incorrect** because it corresponds to an x-coordinate of $x = 20$ at the vertex and a subsequent computation of the value using the *incorrect* function $P(x) = 0.5x^2 + 40x - 300$, whereby the value would be $P(20) = 700$.

8. Ms. Johnston is a sales associate at a jewelry store. Her total weekly earnings consist of a wage of $10 per hour plus a 10 percent commission on her total sales for the week. One week Ms. Johnston worked 30 hours and had total sales of x dollars. Which of the following represents her total weekly earnings y, in dollars, for that week?

A. $y = 10x + 0.01$

B. $y = 0.1x + 300$

C. $y = 0.1(x + 300)$

D. $y = 30(0.1x + 10)$

Answer

Option B is correct. Ms. Johnston's total earnings are $10 times the number of hours worked, plus 10% of her total sales: $y = 10(30) + \frac{10}{100}x = 0.1x + 300$. **Option A is incorrect** because x represents the total sales, not the number of hours worked. **Option C is incorrect** because this equation results from applying the 10% commission after adding the value of sales and the $300 base earnings. **Option D is incorrect** because this equation results from multiplying the 30 hours by the $10-per-hour base pay rate and the 10% commission.

Competency 005—The teacher understands and uses linear functions to model and solve problems.

9. Which of the following is the equation of the line in the *xy*-plane that passes through the points (–7, –2) and (–2, –7)?

A. $x + y = -9$

B. $x - y = -9$

C. $x - y = -5$

D. $-x + y = -5$

Answer

Option A is correct because the values (x, y) of both of the ordered pairs satisfy the equation; that is, the graph of the equation passes through the points in the *xy*-plane. **Option B is incorrect** because neither of the ordered pairs satisfies the equation; that is, the graph of the equation does not pass through either point in the *xy*-plane. **Option C is incorrect** because the values (–2, –7) do not satisfy the equation; that is, the graph of the equation does not pass through that point in the *xy*-plane. **Option D is incorrect** because the values (–7, –2) do not satisfy the equation; that is, the graph of the equation does not pass through that point in the *xy*-plane.

10. In the *xy*-plane, line segment *AB* is bisected by line segment *CD*, and the coordinates of the point of intersection are (–2, –3). If the coordinates of *A* are (–8, –1), what are the coordinates of point *B* ?

A. (5, –5)

B. (4, –5)

C. (–5, –2)

D. (8, 1)

Answer

Option B is correct because (–2, –3) is the midpoint of segment *AB*, meaning that point *B* must lie on the line that connects the two points (–2, –3) and (–8, –1), and the distance from *B* to (–2, –3) is the same as the distance from *A* to (–2, –3). The line $y = -\frac{1}{3}x - \frac{11}{3}$ is the line that contains *A* and (–2, –3), and thus also contains *B*. The distance between the point *A* and (–2, –3) is $2\sqrt{10}$. The equation of the set of points that lies at a distance of $2\sqrt{10}$ from (–2, –3) is $(x + 2)^2 + (y + 3)^2 = 40$. Solving the system of equations, $y = -\frac{1}{3}x - \frac{11}{3}$ and $(x + 2)^2 + (y + 3)^2 = 40$, produces the solutions (4, –5) and (–8, –1). **Options A, C and D are incorrect** because they do not lie at the same distance from (–2, –3) as (–8, –1).

11. A restaurant charges $9.00 for a large pizza and $1.75 for each topping selected. Let *C* be the total cost of a pizza and let *t* be the number of toppings selected. Which of the following is an equation for the total cost *C* of a pizza?

A. $C = 9.00 + 1.75t$

B. $C = (1.75 + 9.00)t$

C. $C = 9.00t + 1.75$

D. $C = (1.75)(9.00)t$

Answer

Option A is correct because if a customer orders t toppings, the total charge will be 1.75t$ plus the base cost of $9.00. **Option B is incorrect** because the multiplier of t should only be used with the cost of each topping. **Option C is incorrect** because $9.00 is the base cost of the pizza; it should not be multiplied by the number of toppings ordered. **Option D is incorrect** because the individual costs should be added, not multiplied.

Competency 006—The teacher understands and uses nonlinear functions and relations to model and solve problems.

12. Which of the following points is the vertex of the graph of $y = 2x^2 - 8x + 1$ in the *xy*-plane?

A. (2,13)

B. (0,1)

C. (4,1)

D. (2,–7)

Answer

Option D is correct because the vertex form of a parabola is $y = a(x - h)^2 + k$ where the coordinates of the vertex are (h, k). The equation can be rewritten as $y = 2(x - 2)^2 - 7$; therefore, the vertex of the graph is the point (2,–7). **Options A, B and C are incorrect** because they result from mistakes in calculation of the vertex form of the equation.

13. Which of the following values of x satisfies $2x^2 + 5x - 3 < 0$?

A. –3

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. 2

Answer

Option B is correct. The function $f(x) = 2x^2 + 5x - 3$ is equal to zero when $x = -3$ or $x = \frac{1}{2}$. The value of f is less than zero for all x such that $-3 < x < \frac{1}{2}$. Of the options listed, only $\frac{1}{3}$ is greater than –3 and less than $\frac{1}{2}$.

Options A and C are incorrect because $f(x) = 0$ when $x = -3$ or $x = \frac{1}{2}$. **Option D is incorrect** because $f(x) > 0$ when $x = 2$.

14. An eighth-grade mathematics teacher is preparing a lesson about exponential decay and plans to use an example involving half-life. The teacher explains that half-life is the amount of time required for the initial quantity of a substance to reduce by half, and the teacher then gives an example of a radioactive substance with a half-life of 40 years. If the initial quantity of the radioactive substance is P grams, what is the quantity, in grams, that will remain after 200 years?

A. $\left(\frac{1}{32}\right)P$

B. $\left(\frac{1}{16}\right)P$

C. $\left(\frac{1}{5}\right)P$

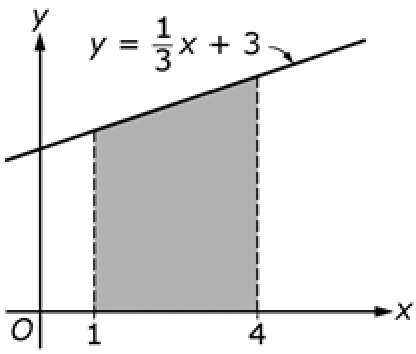
D. $5P$

Answer

Option A is correct because if the initial quantity is P grams, $\left(\frac{1}{2}\right)P = 0.5P$ will remain after 40 years. In 200 years there are $\frac{200}{40}$, or five, 40-year periods. The quantity remaining after five 40-year periods will be $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)P = \left(\frac{1}{2^5}\right)P = \left(\frac{1}{32}\right)P$ grams. **Option B is incorrect** because the quantity $\left(\frac{1}{16}\right)P$ grams is what will remain after only four half-life periods. **Option C is incorrect** because $\left(\frac{1}{5}\right)P$ is $\left(\frac{1}{5}\right)$ of the original amount. **Option D is incorrect** because the quantity of the substance will decrease as it decays exponentially, not increase.

Competency 007—The teacher uses and understands the conceptual foundations of calculus related to topics in middle school mathematics.

Use the figure below to answer the question that follows.



15. A geometry teacher developed a lesson that incorporates solving linear equations using algebra and finding the area of geometric shapes using geometry. Which of the following calculus topics could be demonstrated by finding the area of the trapezoid above?

A. The derivative of a function at a point

B. The definite integral

C. The limit of a function of *x* as *x* goes to infinity

D. Newton's method to find the zeros of a function

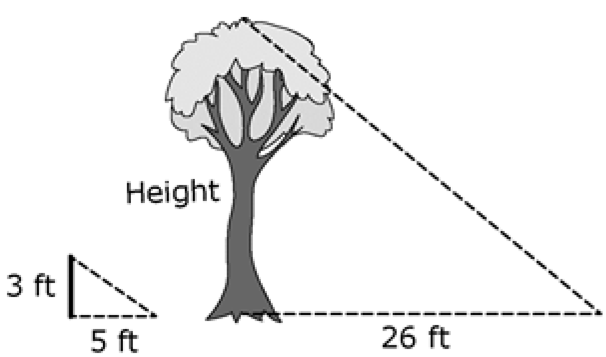
Answer

Option B is correct because the definite integral is the area of the region in the *xy*-plane between the *x*-axis and the graph of a function over a given interval, where the region in this case is a trapezoid. **Option A is incorrect** because the derivative of a function at a point is the instantaneous rate of change of the function at the given point. **Option C is incorrect** because the limit of a function as *x* goes to infinity, if it exists, is a value *L* that the functional values of *f*(*x*) are arbitrarily close to as *x* is arbitrarily large. **Option D is incorrect** because Newton's method is used to find a zero of a function and not the area.

Domain III—Geometry and Measurement

Competency 008—The teacher understands measurement as a process.

Use the figure below to answer the question that follows.



Note: Figure not drawn to scale.

16. At a certain time of day, a student measured the height of the shadow of a yardstick, held vertically, to be 5 feet. At the same time of day, the student measured the length of the shadow of the tree to be 26 feet. To the nearest foot, what is the height of the tree?

A. 16 feet

B. 24 feet

C. 30 feet

D. 43 feet

Answer

Option A is correct because a line drawn from the top of the yardstick to the end of the shadow forms the same angle as a line drawn from the top of the tree to the end of its shadow. Therefore, these two triangles are similar by angle-angle-angle similarity. One of the proportions that follows from similarity is $\frac{3}{5} = \frac{h}{26}$, where *h* is the height of the tree. Solving for *h* yields *h* = 15.6, which, rounded to the nearest foot, is approximately 16 feet. **Option B is incorrect** because it is erroneously obtained from 26 − 5 = 21 and 21 + 3 = 24. **Option C is incorrect** because it results from finding the hypotenuse of the larger triangle: $\sqrt{15.6^2 + 26^2} \approx 30.$ **Option D is incorrect** because it results from the incorrect proportion $\frac{3}{5} = \frac{26}{h}$.

17. Frank completed a 400-meter race in 75 seconds. Which of the following is closest to Frank's speed in kilometers per hour?

A. 17 kilometers per hour

B. 19 kilometers per hour

C. 21 kilometers per hour

D. 23 kilometers per hour

Answer

Option B is correct because $\frac{\text{400 meters}}{\text{75 seconds}} \times \frac{\text{3600 seconds}}{\text{1 hour}} \times \frac{\text{1 kilometer}}{\text{1000 meters}}$ = 19.2 kilometers. Among the options presented, 19 kilometers per hour is the closest to the exact value. **Options A, C, and D are incorrect** because 19 kilometers per hour is closest to the actual value of 19.2 kilometers per hour.

18. Tony purchased 100 kilograms of decorative stones. Each stone weighs approximately 10 grams. Which of the following is the best estimate of the number of stones purchased?

A. 10^3

B. 10^4

C. 10^5

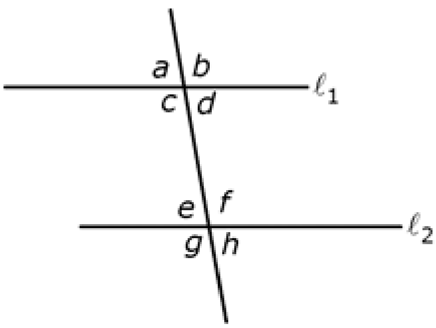
D. 10^6

Answer

Option B is correct because there are 1000 grams in a kilogram, so $100 \times 1000 = 10^2 \times 10^3 = 10^{2+3} = 10^5$ grams of stones were purchased. Since each stone weighs approximately 10 grams, about $\frac{10^5}{10^1} = 10^{5-1} = 10^4$ stones were purchased. **Option A is incorrect** because $10^3 = 1000$ is the number of grams in a kilogram. **Option C is incorrect** because $10^5 = 100{,}000$ is the total number of grams of stones purchased. **Option D is incorrect** because $10^6 = 1{,}000{,}000$ is the total number of grams of stones purchased times the weight per stone.

Competency 009—The teacher understands the geometric relationships and axiomatic structure of Euclidean geometry.

Use the figure below to answer the question that follows.



19. In the diagram above, ℓ_1 is parallel to ℓ_2. If the measure of angle *b* is 100°, what is the measure of angle *e*?

A. 100°

B. 95°

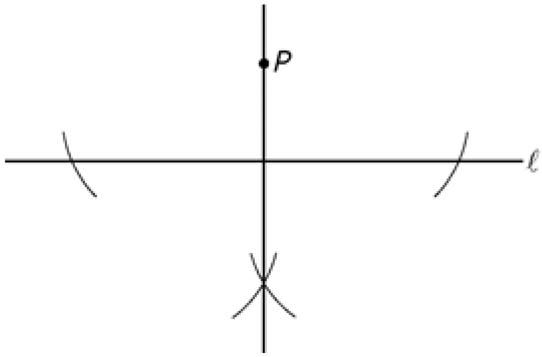
C. 80°

D. 75°

Answer

Option C is correct. Based on the figure, angles *a* and *b* are supplementary, so the sum of their measures is 180°. Since the measure of angle *b* is 100°, the measure of angle *a* is 80°. Because ℓ_1 and ℓ_2 are parallel lines cut by a transversal, corresponding angles are congruent and the measures are equal. Therefore, the measure of angle *a* is equal to the measure of angle *e*, and the measure of angle *e* is thus also 80°. **Options A, B and D are incorrect** because the measure of angle *e* is 80°.

Use the figure below to answer the question that follows.



20. Which of the following describes the geometric construction above, where the construction uses only a compass and a straightedge?

A. The locus of points that are equidistant from line ℓ and point *P*

B. The line perpendicular to line ℓ and passing through point *P*

C. The extension of line ℓ

D. The line parallel to line ℓ and passing through point *P*

Answer

Option B is correct because the arcs that intersect line ℓ at two points, call them *A* and *B*, appear to be part of a circle centered at point *P*, and the two other arcs that intersect each other at a point (call it *C*) appear to be from two circles centered at *A* and *B* with equal radii. If *A*, *B*, and *C* are constructed with a compass as they appear to be, then $PA = PB$ and $CA = CB$. The line passing through points *P* and *C* is then drawn with a straightedge, intersecting ℓ at a point (call it *D*) and the two equalities yield two congruent triangles, *APC* and *BPC*. From this congruence, it follows that triangles *APD* and *BPD* are congruent. Finally, it follows that the four angles at *D* are right angles, and line *PC* is a line that is perpendicular to ℓ and passes through *P*. **Option A is incorrect** because the locus of points equidistant from a line and a point that is not on the line is a parabola, which does not appear in the figure. **Option C is incorrect** because line ℓ is given. **Option D is incorrect** because there are no parallel lines in the figure.

21. Let *ABC* be a triangle, where *AB* has length 4 and *BC* has length 8. For which of the following possible lengths of *AC* is *ABC* an obtuse triangle?

Select <u>all</u> that apply.

A. 6

B. 7

C. 8

D. 9

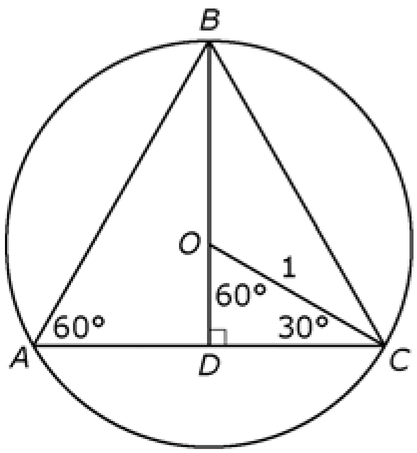
E. 10

Answer

Options A, D and E are correct because a triangle has an obtuse angle if, and only if, the square of the longest length is greater than the sum of the squares of the two shorter lengths. This is true because when "greater than" is replaced by "equal to," the triangle is a right triangle, using the Pythagorean theorem. The inequalities for options A, D and E are $8^2 > 4^2 + 6^2$, $9^2 > 4^2 + 8^2$, and $10^2 > 4^2 + 8^2$, respectively. **Option B is incorrect** because $8^2 < 4^2 + 7^2$. **Option C is incorrect** because $8^2 < 4^2 + 8^2$.

Competency 010—The teacher analyzes the properties of two- and three-dimensional figures.

Use the figure below to answer the question that follows.



22. Equilateral triangle *ABC* is inscribed in a circle with center *O* and a radius of 1, as shown above. The height of the triangle is *BD*. What is the area of triangle *ABC*?

A. $\frac{\sqrt{3}}{2}$

B. $\frac{\sqrt{3}}{8}$

C. $\frac{3\sqrt{3}}{2}$

D. $\frac{3\sqrt{3}}{4}$

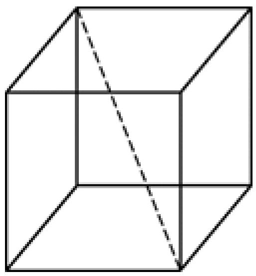
Answer

Option D is correct because the area of a triangle is $\frac{1}{2}bh$, where *b* is the base and *h* is the height. In the triangle shown, *AC* is the base and *BD* is the height. Since triangle *ABC* is an equilateral triangle, angles *A*, *B* and *C* all have measures of 60°. The radius *OC* bisects angle *BCA*, so the measure of angle *OCD* is 30°. Thus, triangle *OCD* is a 30-60-90 triangle, and the length of *OC* is 1 because *OC* is a radius of the circle centered at *O*. Therefore, the length of *OD* is $\frac{1}{2}$ and the length of *DC* is $\frac{\sqrt{3}}{2}$. Thus, the length of *AC*, the base of the triangle, is

$(2)\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3},$ the length of *BD*, the height, is $1 + \frac{1}{2} = \frac{3}{2},$ and the area is $\left(\frac{1}{2}\right)(\sqrt{3})\left(\frac{3}{2}\right) = \frac{3\sqrt{3}}{4}.$

Options A, B and C are incorrect because they are not areas of the triangle *ABC*.

Use the cube below to answer the question that follows.



23. In the cube shown above, a student measured the length of a diagonal to be 4.5 centimeters. Which of the following is the best estimate of the volume of the cube?

A. 121.5 cubic centimeters

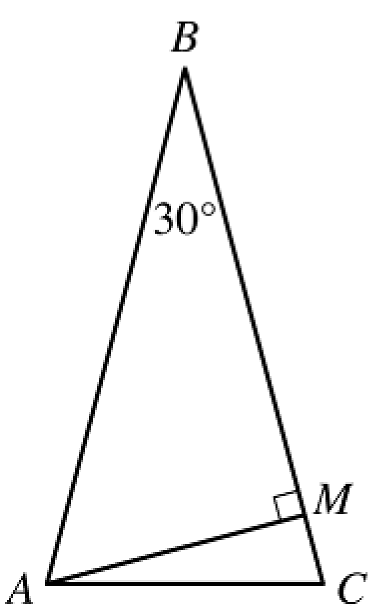
B. 91.1 cubic centimeters

C. 17.5 cubic centimeters

D. 2.6 cubic centimeters

Answer

Option C is correct because the volume *V* of a cube with edges of length *s* is s^3. The diagonal has length $4.5 = \sqrt{3s^2}$, and thus, $s = \frac{4.5}{\sqrt{3}}$. Therefore, $V = \left(\frac{4.5}{\sqrt{3}}\right)^3 \approx 17.5.$ **Option A is incorrect** because it is the surface area of the cube, where 4.5 is the length of an edge. **Option B is incorrect** because it is approximately the volume of the cube, where the length of an edge is 4.5. **Option D is incorrect** because it is approximately the length of an edge of the cube.



24. In triangle *ABC* shown, *AB* = *BC*, the measure of angle *ABC* is 30 degrees, and line segment *AM* is perpendicular to side *BC*. What is the degree measure of angle *MAC*?

A. 10°

B. 15°

C. 60°

D. 75°

Answer

Option B is correct. Since *AB* = *BC*, the triangle shown, *ABC*, is isosceles, and therefore the measure of angle *BAC* must be equal to the measure of angle *BCA*. The sum of the measures of angles *ABC*, *BAC*, and *BCA* must be 180°. Let x be the measure of angle *BAC;* then x is also the measure of angle *BCA,* and $30° + 2x° = 180°$. Thus, $x = \frac{180 - 30}{2} = 75°$. Triangle *AMB* is a right triangle, so the measure of angle *BAM* is 90° − 30° = 60°. The measure of angle *BAC* is the sum of the measures of angles *BAM* and *MAC*, so the measure of angle *MAC* is 75° − 60° = 15°. **Options A, C, and D are incorrect** because the measure of the angle is 15°.

25. A circle with radius r has a circumference of 48. What is the value of r?

A. $r = \frac{24}{\pi}$

B. $r = \sqrt{\frac{48}{\pi}}$

C. $r = \sqrt[3]{\frac{36}{\pi}}$

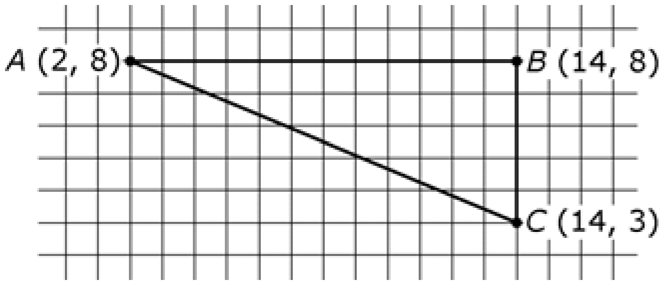
D. $r = \frac{48}{\pi}$

Answer

Option A is correct. The circumference C of a circle of radius r is $C = 2\pi r$. It is given that $C = 48$, therefore $2\pi r = 48$ and $r = \frac{48}{2\pi} = \frac{24}{\pi}$. **Option B is incorrect** because $r = \sqrt{\frac{48}{\pi}}$ is obtained by incorrectly using the formula for the area of a circle, $\pi r^2 = 48$. **Option C is incorrect** because $r = \sqrt[3]{\frac{36}{\pi}}$ is obtained by incorrectly using the formula for the volume of a sphere, $\frac{4}{3}\pi r^3 = 48$. **Option D is incorrect** because $r = \frac{48}{\pi}$ is obtained by incorrectly using r for the diameter of the circle instead of the radius, $\pi r = 48$.

Competency 011—The teacher understands transformational geometry and relates algebra to geometry and trigonometry using the Cartesian coordinate system.

Use the figure below to answer the question that follows.



26. Triangle *A′B′C′* has a hypotenuse of length 52 and is a dilation of triangle *ABC* shown above. What is the scale factor used to dilate triangle *ABC* to transform it to triangle *A′B′C′*?

A. $\frac{1}{4}$

B. $\frac{4}{13}$

C. $\frac{13}{4}$

D. 4

Answer

Option D is correct because, using the Pythagorean theorem, the hypotenuse of triangle *ABC* is 13, and since the hypotenuse of *A′B′C′* is 52, it follows that triangle *ABC* has been dilated by a factor of $\frac{52}{13}$, or 4. **Option A is incorrect** because it is equal to the reciprocal of $\frac{52}{13}$. **Option B is incorrect** because it assumes that the hypotenuse of triangle *ABC* is 13^2, or 169, in which case the dilation factor would be $\frac{52}{169}$, or $\frac{4}{13}$. **Option C is incorrect** because it assumes that the hypotenuse of triangle *ABC* is 13^2, or 169, and then uses the reciprocal of what would be the dilation factor, that is, the reciprocal of $\frac{52}{169}$, or $\frac{13}{4}$.

Domain IV—Probability and Statistics

Competency 012—The teacher understands how to use graphical and numerical techniques to explore data, characterize patterns and describe departures from patterns.

Use the list below to answer the question that follows.

90, 70, 60, 75, 80, 82, 85, 88, 80, *x*

27. The list above shows ten scores from a recent test in a math class. The range of the ten scores is 30, and the interquartile range is 10. Which of the following could be the value of *x* ?

A. 74

B. 84

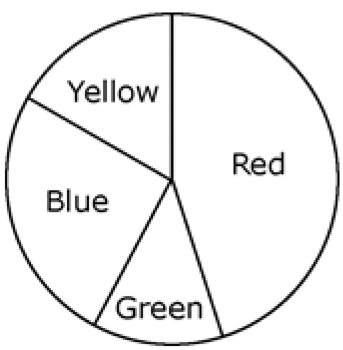
C. 86

D. 96

Answer

Option B is correct. The range of the numbers in a data set is the difference between the greatest number and the least number in the data set. The quartiles are obtained by ordering the data from the least value to the greatest value and then dividing the data into four equal groups. The interquartile range is the difference between the third quartile and the first quartile. Using the value of 84 from option B and ordering the list from least to greatest—60, 70, 75, 80, 80, 82, 84, 85, 88, 90—the range is 90 − 60 = 30, the first quartile is 75, the third quartile is 85, and the interquartile range is 85 − 75 = 10. **Option A is incorrect because** using the value of 74 from option A and ordering the list from least to greatest—60, 70, 74, 75, 80, 80, 82, 85, 88, 90—the range is the first quartile is 74, and the third quartile is 85, so the interquartile range is 85 − 74 = 11. **Option C is incorrect** because using the value of 86 from option C and ordering the list from least to greatest—60, 70, 75, 80, 80, 82, 84, 85, 86, 88, 90—the range is 90 − 60 = 30, the first quartile is 75, and the third quartile is 86, so the interquartile range is 86 − 75 = 11. **Option D is incorrect** because using the value of 96 from option D and ordering the list from least to greatest—60, 70, 75, 80, 80, 82, 85, 88, 90, 96—the range is 96 − 60 = 36.

Use the circle graph below to answer the question that follows.



28. Ms. Jefferson read an article to her class describing a survey of students who were asked to choose their favorite color among the colors yellow, blue, green, and red. The graph above shows the results of the survey. Ms. Jefferson tells the class that the survey results are representative of all students, and she asks the class to predict how many of the 850 students in their school would choose either yellow or green. Which of the following is the best estimate?

A. 20

B. 25

C. 150

D. 250

Answer

Option D is correct because the sectors in the circle graph can be estimated as follows. The sector for blue has a central angle that is very close to a right angle, which is about 25% of the circle, and the sector for red is somewhat less than half the circle, perhaps about 45% of the circle. Thus, blue and red together constitute about 25% + 45%, or 70%, of the data. It follows that yellow and green together constitute about 30% of 850, or 255 students. Among the options, 250 is closest to 255. **Option A is incorrect** because it is at least one order of magnitude less than the estimate given in option D. **Option B is incorrect** because it is at least one order of magnitude less than the estimate given in option D. **Option C is incorrect** because the estimate of 150 is about 18% of 850; therefore, if blue is about 25% of the data, then red could be greater than 50% of the data, which would contradict the graph.

29. The height of each student at Jefferson Middle School was measured and recorded. Joseph was told that his height was at the 60^{th} percentile of the heights of the students. Which of the following must be true?

A. The heights of 4 students are greater than Joseph's height.

B. Joseph's height is 60 inches.

C. The heights of at least 60 percent of the students are less than or equal to Joseph's height.

D. If the height of the tallest student is 70 inches, then Joseph's height is (0.6)(70) = 48 inches.

Answer

Option C is correct because a percentile is a measure used in statistics that indicates the value below which a given percent of observations in a group of observations fall. Joseph's height at the 60^{th} percentile of the heights of students means that the heights of at least 60 percent of the students are less than or equal to Joseph's height. **Option A is incorrect** because the total number of students is not given. **Option B is incorrect** because none of the heights are given. **Option D is incorrect** because being at the 60^{th} percentile of the heights measured does not mean that Joseph's height is 60 percent of the height of the tallest student.

List L: 4, 9, 4, 8, 9, 8, 11, 10, 9

30. List L is shown above. Let $\bar{x}$ be the mean, let m be the median, and let D be the mode of the numbers in list L. Which of the following is true?

A. $\bar{x} = m$ and $m = D$

B. $\bar{x} = m$ and $m < D$

C. $\bar{x} < m$ and $m = D$

D. $\bar{x} < m$ and $m < D$

Answer

Option C is correct. The mean of the numbers in the list is the sum of the numbers divided by the number of numbers in the list, $\bar{x} = \frac{4 + 9 + 4 + 8 + 9 + 8 + 11 + 10 + 9}{9} = \frac{72}{9} = 8$. To find the median, arrange the numbers in list L in increasing order: 4, 4, 8, 8, 9, 9, 9, 10, 11. There are 9 numbers in the list. The median is the number in the middle of the list, that is, the fifth number, $m = 9$. The mode of the numbers in a list is the number in the list that appears the most often: in this case $D = 9$. Thus, $\bar{x} < m$ and $m = D$. **Options A and B are incorrect** because $\bar{x} \neq m$. **Option D is incorrect** because $m = D$.

31. To form a committee, a principal will choose 3 students from a group of 5 seventh-grade students and 2 students from a group of 6 eighth-grade students. What is the total number of different committees the principal could select?

A. 16

B. 150

C. 180

D. 720

Answer

Option B is correct because the number of combinations of *n* objects taken *k* at a time is given by the formula $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. If the principal chooses 3 students from a group of 5 seventh-grade students and 2 students from a group of 6 eighth-grade students, the number of possible different committees is $\binom{5}{3}\binom{6}{2} = \frac{5!}{3!2!}\frac{6!}{2!4!} = \left(\frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1)(2 \cdot 1)}\right)\left(\frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1)(4 \cdot 3 \cdot 2 \cdot 1)}\right) = (10)(15) = 150.$ **Options A, C and D are incorrect** because the calculations are not correct.

32. Six swimmers are competing in a 25-meter race. If there are no ties, how many different combinations are possible for a first-, second-, and third-place finish?

A. 216

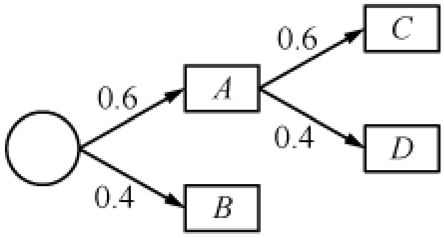
B. 120

C. 18

D. 15

Answer

Option B is correct because it is possible for any of the 6 swimmers to finish first, which leaves 5 possible second-place finishers and 4 possible third-place finishers. Using the multiplication rule, there are 6 × 5 × 4 = 120 possible combinations. **Option A is incorrect** because this is the result of 6^3, which does not take into consideration that the swimmer who finishes first cannot finish second or third, and the swimmer who finishes second cannot finish third. **Option C is incorrect** because this is the result of 6 × 3. **Option D is incorrect** because this is the result of 6 + 5 + 4.



33. Mr. Garcia showed his mathematics class the probability tree diagram shown above, where 0.6 and 0.4 represent the probability that an event will occur. Based on the diagram, what is the probability that event *D* will occur?

A. 1.00

B. 0.36

C. 0.24

D. 0.20

Answer

Option C is correct. Probability tree diagrams can be used to represent a series of independent events. Each node on the diagram represents an event and the probability that the event will occur is shown. The parent node represents a certain event and has probability 1. Each set of subsequent nodes represents an exclusive and exhaustive partition of the parent event. The probability associated with a node is the probability of that event occurring after the parent event occurs. For example, in the tree diagram shown, the probability that event *A* will occur is 0.6. The probability that the series of events leading to a particular node will occur is equal to the product of the probability that node will occur times the probabilities of the occurrence of the preceding events. In the diagram shown, the probability that event *D* will occur is the product of the probability that event *A* will occur times the probability that event D will occur, $P(D) = (0.6)(0.4) = 0.24$. **Option A is incorrect** because 1.00 is the sum of 0.6 and 0.4, while the probability that event D will occur is the product of 0.6 and 0.4. **Option B is incorrect** because 0.36 is the product of 0.6 and 0.6, which is the probability that event C will occur. **Option D is incorrect** because the probability that event D will occur is the product (0.6)(0.4) and not the difference, 0.6 − 0.4.

34. A high school issues each new student a 7-character identifier. The first 3 characters are the first 3 letters in the student's last name, all capital letters from the 26-character English alphabet. The 3 letters are followed by 4 randomly generated integers from 0 to 9, inclusive. Repetition of letters and integers is allowed. How many unique identifiers can be generated?

A. $(26^3)(10^4)$

B. $(26)(25)(24)(10^4)$

C. $(26)(25)(24)(10)(9)(8)(7)$

D. $\dfrac{(26!)(10!)}{(3!)(23!)(4!)(6!)}$

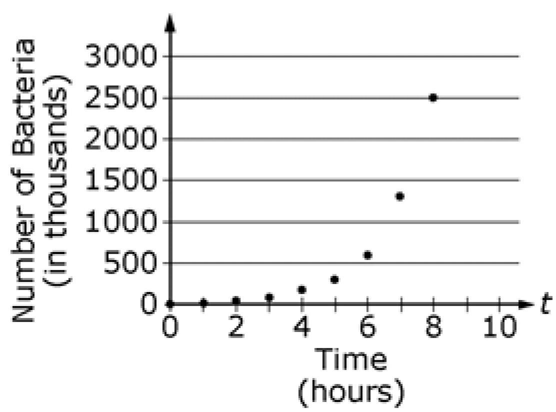
Answer

Option A is correct. There are 26 possible letters that can be chosen for the first letter in the 7-character identifier. Since repetition is allowed, there are also 26 possible choices for the second and third letters. Then the number of possible letter combinations is 26^3. Similarly, since repetition is allowed for the integer characters, the number of possible combinations of the 4 integers in the identifier is 10^4. The total number of unique identifiers is thus $(26^3)(10^4)$. **Option B is incorrect** because (26)(25)(24) is the number of possible 3-letter combinations if repetition is <u>not</u> allowed. **Option C is incorrect** because (26)(25)(24)(10)(9)(8)(7) is the number of 7-character identifiers if repetition is <u>not</u> allowed for either the letters or the integers. **Option D is incorrect** because $\frac{(26!)(10!)}{(3!)(23!)(4!)(6!)}$ is the number of combinations of 26 letters taken 3 at a time multiplied by the number of combinations of 10 integers taken 4 at a time.

Competency 014—The teacher understands the relationship among probability theory, sampling and statistical inference and how statistical inference is used in making and evaluating predictions.

Use the information below to answer the question that follows.

Time t (hours)	Number of Bacteria (in thousands)
0	10
1	18
2	35
3	85
4	170
5	300
6	600
7	1300
8	2500



35. Every hour, a scientist counted the number of bacteria growing in a certain medium. The scientist recorded the results in a table and produced the scatterplot shown above. If $P(t)$ is a mathematical model for the number of bacteria, in thousands, at time t hours, which of the following expressions is the best fit for $P(t)$?

A. $P(t) = 300t + 10$

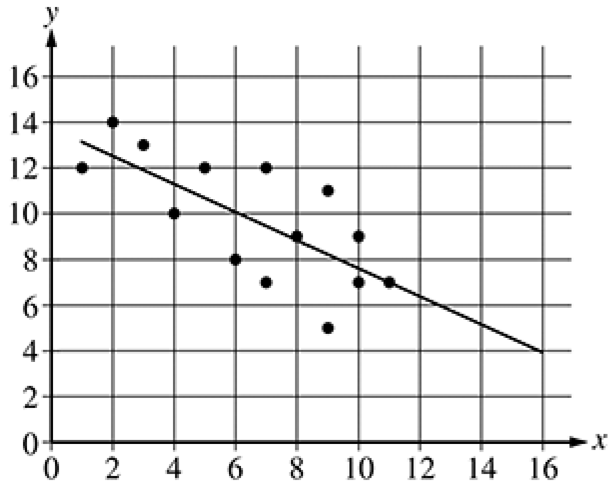
B. $P(t) = 300t^2 + 100t + 10$

C. $P(t) = 10(2^t)$

D. $P(t) = 10(\ln(2t + 1))$

Answer

Option C is correct. Based on the scatterplot, an exponential function is the best model for the data shown. Evaluation of $P(t) = 10(2^t)$ for values of $t \geq O$ shows that this function produces values close to the data recorded. **Option A is incorrect** because the data shown in the scatterplot are not linear. **Option B is incorrect** because a parabola is not the best fit for the data shown in the scatterplot. **Option D is incorrect** because a logarithmic function is not the best fit for the data shown.



36. The scatterplot shows the relationship between two sets of data, *x* and *y*. The line of best fit of the relationship between the two data sets is shown. Based on the line of best fit, which of the following is the best estimate of the value of *y* when the value of *x* is 14?

A. 0

B. 2

C. 5

D. 7

Answer

Option C is correct. Based on the graph of the line of best fit, we can determine that when the value of *x* is 14, then the value of *y* is between 4 and 6. Given the four answer choices, option C, 5, is the best answer. **Options A, B, and D are incorrect** because 5 is the best estimate of the value of *y* when *x* is 14.

37. In 2016 there were 324,582 licensed teachers in a certain state. The ages of the teachers were modeled with a normal distribution. The mean age was 43.5 years old, with a standard deviation of 8.2 years. Based on this distribution, which of the following is the best estimate of the number of licensed teachers in the state in 2016 with an age of 59.9 years old or greater?

A. 8000 teachers

B. 16,000 teachers

C. 55,000 teachers

D. 110,000 teachers

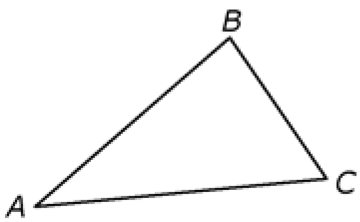
Answer

Option A is correct. The graph of a normal distribution is bell-shaped and symmetric about a vertical line through its center. The mean, median, and mode of a normal distribution are all equal and occur at the center of the distribution. About 68% of all data values of a normal distribution lie within 1 standard deviation of the mean, in both directions. About 95% of the data values lie within 2 standard deviations of the mean, in both directions. Thus, about 2.5% of the data values are greater than 2 standard deviations about the mean. The mean is given to be 43.5 years old with a standard deviation of 8.2 years, and 43.5 + 2(8.2) = 59.9. Therefore, about 2.5% of the 324,582 licensed teachers in the state are 59.9 years old or older, and $\frac{2.5}{100}$(324,582) = 8114 ≈ 8000. **Option B is incorrect** because 16,000 licensed teachers is about 5% of the total number of teachers. **Option C is incorrect** because 55,000 licensed teachers is about 17% of the total. **Option D is incorrect** because 110,000 is about 34% of the total.

Domain V—Mathematical Processes and Perspectives

Competency 015—The teacher understands mathematical reasoning and problem solving.

Use the figure and the proof below to answer the question that follows.



Given: In triangle *ABC* shown, *AB* > *BC*

Prove: ∠*A* ≠ ∠*C*, that is the measure of angle *A* is not equal to the measure of angle *C*.

Proof by contradiction: Assume that ∠*A* ≅ ∠*C*.

If ∠*A* ≅ ∠*C*, then $\overline{AB} \cong \overline{AC}$ by the ______________. However, this theorem contradicts the given information that *AB* > *BC*. Therefore, the assumption that ∠*A* ≅ ∠*C* must be false, and so the measures of the angles cannot be equal and ∠*A* ≠ ∠*C*.

38. In the proof above, which of the following theorems correctly fills in the blank?

A. corresponding angles theorem

B. angle bisector theorem

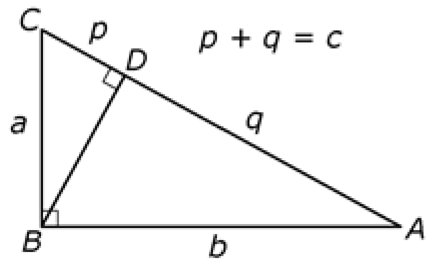
C. isosceles triangle theorem

D. congruence of triangles with the side-side-side property

Answer

Option C is correct because the converse of the isosceles triangle theorem is the statement that if two angles of a triangle are congruent, then the sides opposite the two angles are congruent. **Option A is incorrect** because the corresponding angles theorem involves two parallel lines and a transversal. **Option B is incorrect** because the angle bisector theorem involves an angle that is bisected. **Option D is incorrect** because the side-side-side comparison is used to determine if two triangles are congruent.

Use the figure and the proof below to answer the question that follows.



Statement	Reason
$\frac{p}{a} = \frac{a}{c}$ and $\frac{q}{b} = \frac{b}{c}$	1. ?
$p = \frac{a^2}{c}$ and $q = \frac{b^2}{c}$	2. Multiplication property of equality
$c = p + q = \frac{a^2}{c} + \frac{b^2}{c}$	3. Substitution
$c^2 = a^2 + b^2$	4. Multiplication property of equality

39. A seventh-grade mathematics teacher presented the proof of the Pythagorean theorem shown above, with one missing reason for students to supply. What is the missing reason?

A. Triangles *CDB* and *BDA* are similar.

B. Apply the angle-side-angle theorem to triangle *BDA*.

C. Apply the side-angle-side theorem to triangle *CBA*.

D. Triangles *CBA* and *CDB* are similar, and triangles *CBA* and *BDA* are similar.

Answer

Option D is correct because the similarity of triangle *CBA* and *CDB* implies that the lengths of their corresponding sides have the same ratio, yielding $\frac{p}{a} = \frac{a}{c}$. Similarly, the similarity of triangles *CBA* and *BDA* yields $\frac{q}{b} = \frac{b}{c}$. **Option A is incorrect** because although it is true that triangles *CDB* and *BDA* are similar, *c* is not equal to any of the lengths of the sides of these triangles. **Options B and C are incorrect** because the angle-side-angle and side-angle-side theorems refer to the congruence of triangles.

Competency 016—The teacher understands mathematical connections within and outside of mathematics and how to communicate mathematical ideas and concepts.

40. A teacher would like to instruct students about semiregular tessellations of a plane. A semiregular tessellation of a plane uses more than one type of regular polygon. Also, every vertex in the tessellation has the same arrangement of polygons around it, where the sum of the angles around the vertex is 360°. The teacher proposes a tessellation that uses three types of regular polygons: a 15-gon, a triangle, and a third type. What is the third type of regular polygon?

A. A pentagon

B. A hexagon

C. An octagon

D. A decagon

Answer

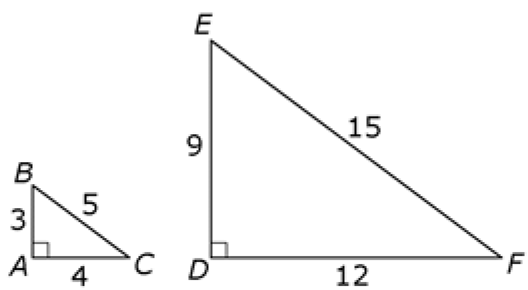
Option D is correct because the measure of an interior angle of a regular 15-gon is $\frac{(15 - 2)(180°)}{15}$, or 156°. When 156° is added to 60°, the measure of an interior angle of an equilateral triangle, the result is 216°. Then 360° − 216° = 144°. The equation $\frac{(n - 2)(180°)}{n}$ = 144° can be used to determine that n = 10, and therefore, the third type of polygon is a decagon. **Option A is incorrect** because an interior angle of a regular pentagon measures 108°, not 144°. **Option B is incorrect** because an interior angle of a regular hexagon measures 120°, not 144°. **Option C is incorrect** because an interior angle of a regular octagon measures 135°, not 144°.

Use the student work below to answer the question that follows.

Student Work

Using the figure below, determine if the following statement is true or false, and explain your reasoning.

Statement: $m\angle C < m\angle F$



(True) or False   Why? Because $\triangle DEF$ is bigger and $\overline{DE}$ is longer than $\overline{AB}$.

41. Which of the following is the most appropriate way for a mathematics teacher to respond to the student work shown?

A. The student's work is correct.

B. The student's calculations are correct, but the teacher should ask the student to calculate the measure of each angle.

C. The student's work is incorrect because the angle measures are not given and cannot be determined from the information provided.

D. The student's work is incorrect because the triangles are similar; therefore, $m\angle C = m\angle F$.

Answer

Option D is correct because the two triangles are similar since the corresponding sides have the same ratio; therefore, the corresponding angles must be congruent. **Options A and B are incorrect** because $\angle C$ and $\angle F$ are, in fact, congruent, so their measures are equal. **Option C is incorrect** because although the student's work is not correct and the angle measures are not given, the angle measures can, in fact, be determined from the information given using inverse trigonometric functions; for example, $m\angle C$ = arcsin($\frac{3}{5}$).

42. Rachel and Peter deposited $25,000 into a new savings account with an annual interest rate of 3.5 percent, compounded semiannually. If they do not make any withdrawals from the account or make any other deposits to the account, which of the following is closest to the balance in the account after three years?

A. $26,335.60

B. $27,625.00

C. $27,742.56

D. $30,731.38

Answer

Option C is correct. If a principal of *P* dollars is invested at an annual interest rate of *r* percent compounded *n* times per year, and no further withdrawals or deposits are made to the account, then the future value *A* of the account balance after *t* years is given by the formula $A = P\left(1 + \frac{r}{100n}\right)^{nt}$. In this problem, *P* is $25,000; *n* is 2 (semiannually means two times per year); *r* is 3.5 percent, which becomes 0.035 in the formula; and *t* is 3 years. When those values are substituted into the formula shown, the future value *A* of the account balance will be $A = \$25{,}000\left(1 + \frac{0.035}{2}\right)^{(2)(3)} = \$27{,}742.56.$ **Option A is incorrect** because the number of compounded periods is (3)(2) = 6, not just 3. **Option B is incorrect** because the interest is compounded; therefore, the interest rate for three years is not the simple interest of (3)(0.035) = 0.105. **Option D is incorrect** because $30,731 = $25,000(1 + 0.35)^6, i.e., the interest is compounded semiannually, not annually.

43. In 2016 Mr. Schuppan paid an annual property tax of $1500. In 2017 Mr. Schuppan paid an annual property tax of $1650. What was the percent increase of Mr. Schuppan's property tax from 2016 to 2017?

A. 8.03%

B. 9.09%

C. 10.0%

D. 15.0%

Answer

Option C is correct. The percent increase of Mr. Schuppan's property tax from 2016 to 2017 is $\frac{1650 - 1500}{1500} \times 100\% = \frac{150}{1500} \times 100\% = 10\%$. **Options A and D are incorrect** because they are obtained by computational errors. **Option B is incorrect** because 9.09% is the percent change with respect to the 2017 tax instead of the 2016 tax.

Domain VI—Mathematical Learning, Instruction and Assessment

Competency 017—The teacher understands how children learn and develop mathematical skills, procedures and concepts.

Use the example below to answer the question that follows.

$$\frac{2x^4 - 3x^3 + x^2 - 4x + 5}{x - 1}$$

44. A teacher is preparing a unit on polynomial long division and would like to use the problem shown above as an example. Before discussing the example, of the following, which concept is best for the teacher to review?

A. Multiplying two rational expressions

B. Vertical asymptotes

C. The additive property of equality

D. The division algorithm for real numbers

Answer

Option D is correct because the division algorithm for real numbers should be familiar to the students. The same algorithm is used in polynomial long division. **Option A is incorrect** because the multiplication of rational expressions is not needed in polynomial long division. **Option B is incorrect** because the unit is about polynomial long division. A vertical asymptote may be a feature of the graph of a rational function in the *xy*-plane and should be discussed in a unit on graphing. **Option C is incorrect** because the additive property of equality is useful for solving equations for a variable but is not necessary in polynomial long division.

$$\frac{36x^3y^{-2}z^4}{18x^2y^{-3}z^6}$$

45. A teacher is preparing a unit about simplifying algebraic expressions and would like to use the expression shown above as an example. Before discussing the example, which of the following concepts is best for the teacher to review?

A. The zero-product property of multiplication

B. Extraneous solutions

C. The laws of exponents

D. The properties of multiplication of real numbers

Answer

Option C is correct because simplifying the expression requires application of the laws of exponents. **Option A is incorrect** because the expression is not set equal to zero; the student is not asked to solve for a variable. **Option B is incorrect** because the student is not asked to solve for a variable, and thus no extraneous solutions will be produced. **Option D is incorrect** because the laws of exponents are the primary group of operations used to simplify the expression.

Competency 018—The teacher understands how to plan, organize and implement instruction using knowledge of students, subject matter and statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) to teach all students to use mathematics.

46. Marcus is renting a bicycle. The rental requires a down payment of $15 plus $6 for each hour the bicycle is rented. If C is the total charge and t is the number of hours that Marcus rented the bicycle, which of the following equations represents the relationship between the amount of time he rented the bicycle and the total cost?

A. $C = \frac{1}{6}t + 15$

B. $C = 6t + 15$

C. $C = 15t + 6$

D. $C = 15(t - 1) + 6$

Answer

Option B is correct because in order to compute the total cost for t hours, the price per hour must be multiplied by the number of hours and added to the down payment, resulting in $C = 6t + 15$. **Option A is incorrect** because the cost per hour in this model would be $\$\frac{1}{6}$ rather than $6. **Options C and D are incorrect** because the cost per hour in this model would be $15 rather than $6.

47. A mathematics teacher assigns students in a sixth-grade class to keep a mathematics diary. Every day, the students are asked to record when and how they use mathematics in their daily lives. At the end of each week, the students each write a short report on their use of mathematics, and the teacher reviews their diaries. Which of the following is the teacher demonstrating with this activity?

A. An understanding of a variety of questioning strategies to encourage mathematical discourse and to help students analyze, evaluate, and communicate their mathematical thinking

B. An understanding of the use of inductive reasoning to make conjectures and deductive methods to evaluate the validity of conjectures

C. An understanding of the purpose, characteristics, and uses of summative assessments

D. An understanding of how technological tools and manipulatives can be used appropriately to assist students in developing, comprehending, and applying mathematical concepts

Answer

Option A is correct because the primary purpose of this activity is to engage the students in mathematical discourse and develop their communication skills. **Option B is incorrect** because inductive reasoning may be used but is not the primary focus. **Option C is incorrect** because this activity is not part of a summative assessment. **Option D is incorrect** because technology and manipulatives are not necessarily used in this activity.

48. Let $f(x) = (x - 100)(x - 125)(x - 150)(x - 175)$. A student wants to use a graphing calculator to find the points in the xy-plane where the graph of f crosses the x-axis. Which of the following bounds on x can be used in a viewing window

so that the calculator displays all the points where the graph of f crosses the x-axis?

A. $-391{,}000 \le x \le 75$

B. $-200 \le x \le -75$

C. $-100 \le x \le 99$

D. $75 \le x \le 200$

Answer

Option D is correct because the zeros of the function are 100, 125, 150, and 175. If the window on the graphing calculator is set to $75 \le x \le 200$, then all the x-intercepts of the graph in the xy-plane will be displayed. **Option A is incorrect** because this window does not include the zeros of the function. The minimum value of the function is –390,625, so –391,000 is an appropriate value for the lower bound of the y values. **Options B and C are incorrect** because these ranges do not include the zeros of the function on the x-axis.

Competency 019—The teacher understands assessment and uses a variety of formal and informal assessment techniques to monitor and guide mathematics instruction and to evaluate student progress.

49. A mathematics teacher finished a unit on adding fractions and gave a formative assessment. While grading the assessment, the teacher found that more than half of the students were making the error $\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$. Which of the following is the best way for the teacher to address this error?

A. Reviewing fraction addition and giving another formative assessment with questions designed to determine common errors among the students

B. Moving to the next section in the text and including questions on fraction addition on the summative review at the end of the semester

C. Asking the students to perform a search of newspapers and Web sites and document how fractions are used in news stories

D. Moving to the next section in the text and assigning extra-credit homework on fraction addition

Answer

Option A is correct because the formative assessment showed that students need additional instruction and practice in adding fractions. Formative assessments can be designed to determine common errors and misconceptions. **Options B and D are incorrect** because students should be able to add fractions correctly before learning new material. **Option C is incorrect** because, while newspaper and Web site searches are interesting and helpful, they do not address the issue of fraction addition.

50. A mathematics teacher finishes a unit on multiplying expressions with exponents and gives a formative assessment. While grading the assessment, the teacher finds that more than half of the students made the error $(2^4)(2^5) = 2^{20}$. Which of the following is the best description of the error?

A. The students multiplied the exponents instead of adding the exponents.

B. The students should have expanded $2^4 = 16$ and $2^5 = 32$ then multiplied the numbers instead of using the properties of exponents.

C. The students should not have omitted the multiplication sign between the two numbers in parentheses.

D. The students should have expanded the answer, 2^{20}, to an integer.

Answer

Option A is correct because $(2^4)(2^5) = 2^{4+5} = 2^9$. **Options B, C, and D are incorrect** because $(a^b)(a^c) = a^{b+c}$ not a^{bc}.